Do Public Asset Purchases Foster Liquidity?*

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Abstract

We analyze how public asset purchases affect liquidity and risk sharing in complete but concentrated financial markets. Even "neutral" policies, such as budget-balanced trades of risk-free debt, alter private portfolios and have distributional effects. Public debt *purchases* worsen liquidity and risk sharing, while debt issuance improves risk sharing but distorts inter-temporal trade. Targeted interventions induce limited spillovers to substitute assets even in integrated markets. Based on these mechanisms, we derive quantity-based trading rules for efficient public portfolio management that depend on the types of assets purchased. A calibration to portfolio data from a Eurozone asset-purchase program illustrates our findings.

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1 Introduction

Central banks around the world now commonly buy and sell securities in financial markets to achieve a variety of policy objectives. Although these tools were initially deployed to stem financial crises and circumvent bounds on short-term policy rates, they have since become established parts of the policy toolkit even outside of crises. As result, central banks around the world now hold, and dynamically manage, large balance sheets to achieve certain rates of return and foster market functioning even in normal times.[1] In this paper, we provide a framework to analyze how public asset trading affects market liquidity and risk sharing among financial institutions operating in imperfectly liquid markets, and how these effects depend on the types of assets traded by central banks.

A common prevailing view is that central bank asset purchases tend to foster liquidity by raising asset prices that would otherwise be depressed because of, e.g., fire sales or other financial constraints. However, this mechanism does not necessarily apply in "normal times" where such constraints are less likely to binding, and it is not obvious how it might extend to the wide variety of asset classes that are now routinely part of central bank operations. Indeed, given that government trading now frequently occurs outside of crises, policymakers have also expressed concern that public asset trading may be distortionary and harm market functioning in normal times (see, among others, speeches and reports by Bernanke (2012), Coeurè (2015) and Logan and Bindseil (2019)).

There is growing empirical support for this view. Wallen and Stein (2023) show that price elasticities in Treasury markets are high when Treasury bills are scarce, while Pinter and Walker (2023) find that financial institutions reduce their hedging of interest rate risk when central banks engage in expansionary polices. Closely related to our proposed mechanism, Pinter, Siriwardane, and Walker (2024) show that strategic liquidity providers such as hedge funds *chose* to time the bottom of the fire sale in Gilt bonds during the LDI crisis to extract price concessions, thereby making it harder for other market participants to hedge interest rate risk. Similarly, Breckenfelder, Collin-Dufresne, and Corradin (2024) document imperfect competition in bond markets during ECB bond pr-

[1] In the U.S., the management of interest rate was the main motivation for quantitative easing and large-scale asset purchase programs of government bonds. More recently, the Federal Reserve's Secondary Market Corporate Credit Facility was introduced to improve corporate bond liquidity. In Japan, the Bank of Japan uses stock purchases to manage the equity premium and has become the largest holder of Japanese equities in the world. In Europe, the ECB has extensively purchased both sovereign and corporate bonds, among other assets.

chase programs. Pelizzon, Subrahmanyam, and Tomio (2022) further show that central bank liquidity interventions broadly distort private liquidity provision and portfolios.

To assess the underlying economic mechanisms and to derive policy implications, we develop a model of risk sharing in which financial markets are endogenously illiquid. The only impediment to trade is the empirically relevant concern of price impact and strategic liquidity provision in concentrated markets, as documented for European and UK bond markets by Pinter, Siriwardane, and Walker (2024) and Breckenfelder, Collin-Dufresne, and Corradin (2024).[2] This yields a tractable framework in which central bank purchases can affect asset prices and market liquidity, but investors still choose privately optimal portfolios in an unconstrained manner. This allows us to derive policy lessons for liquidity management outside of crises.

Our main finding is that endogenous illiquidity leads assets to be overpriced in normal times, reflecting endogenous scarcity and markups rather than fire sale discounts. This reflects the simple empirically revelant intuiton that high-priced assets are those that provide valuable insurance, whereas low-price assets such as stocks provide high returns because they pay off in good states.[3] Hence, public asset purchases that further inflate asset prices harm liquidity and risk sharing, while sales or asset issuance can raise liquidity and improve risk sharing, albeit at the cost of distorting inter-temporal trade. We characterize this mechanism in detail and derive implications for optimal public portfolio management, including a notion of "capacity constraints" beyond which the welfare impact of asset sales is negative.

Trading volumes play the central role in our model. Given imperfect liquidity, investors ration trading quantities to manage their terms of trade. As a result, financial gains from trade are not fully exhausted, leading to inefficient risk sharing and distorted inter-temporal trade. In this context, budget-balanced government trading can affect private portfolios in two ways: directly by influencing asset prices and price impact, and indirectly by altering the distribution of gains from trade. For example, if the government buys assets from an investor in exchange for cash, that investor may need to trade more with other market participants to undo her trade with the government. Under imperfect liquidity, such a budget-balanced transaction is *not* neutral for final allocations

[2] More broadly, Allen and Wittwer (2022) and Hau, Hoffmann, Langfield, and Timmer (2017) document imperfect competition in Treasury and derivatives markets, respectively.

[3] We formally capture this effect by assuming utility functions with positive third derivatives.

even when markets are, in principle, complete.

The non-neutrality of government trading offers a lever to influence risk sharing and inter-temporal smoothing among private market participants, and it can create winners and losers even with purportedly "neutral" policies, such as buying and selling risk-free debt. To isolate these effects, we remove all frictions other than imperfect liquidity: markets are complete and integrated, the government fully funds expenditures with non-distortionary taxes, and redistributes portfolio payouts to investors lump sum.

We begin by studying the effects of trading risk-free debt on private risk sharing when all gains from trade are across states of the world. Strikingly, we find that government purchases of risk-free debt from the market (which lower interest rates) generically *increase* price impact and worsen risk management by private market participants, while asset sales (which raise interest rates) *improve* private risk sharing arrangements. This suggests that expansionary unconventional monetary policy can have unintended consequences for market liquidity and trading efficiency.

This risk-taking channel is conceptually distinct from the canonical "reach for yield" view of risk-taking under low interest rates, and it is consistent with empirical evidence.[4] Pinter and Walker (2023) show that markets for interest rate risk are concentrated, and that non-bank financial institutions, including pension funds and insurance companies, do not fully hedge interest rate risks in derivatives markets, in particular during monetary expansions. Joyce, Liu, and Tonks (2017) show that U.K. insurance companies and pension funds shifted from Gilts toward less liquid corporate bonds during the Global Financial Crisis because of the Bank of England's quantitative easing program.

Underlying this mechanism is a robust sorting property of optimal portfolio choice under price impact: investors sell claims on a given state if and only if they expect to be relatively wealthy in that state, and they trade off price impact against distortions to their optimal portfolio. Under convex marginal utility (e.g. CRRA), this implies that sellers (who are rich) face a low marginal utility cost of portfolio distortions, while buyers (who are poor) face a high cost. Thus, sellers ration supply more than buyers reduce demand,

[4] According to the reach-for-yield view, financial institutions substitute toward riskier assets when interest rates are low to earn higher returns. In our setting, risk taking instead reflects inefficient diversification and may not lead to higher expected returns. The two mechanisms also have different origins. While reach for yield is driven by portfolio restrictions, high leverage or moral hazard (e.g. Martinez-Miera and Repullo (2017)), our mechanism operates when financial institutions are unconstrained and well-capitalized. Finally, our mechanism depends directly on *quantities* traded, not just on interest rates.

creating endogenous scarcity in every asset. As a result, government asset sales improve risk sharing by alleviating this endogenous scarcity even when markets are complete and the securities traded by the government (i.e., risk-free debt) are orthogonal to risk sharing.

When investors also face inter-temporal gains from trade, the purportedly "neutral" policy of trading risk-free debt also has distributional consequences. In particular, public trading of risk-free debt aligns with the desired direction of trade of some investors, creating winners and losers and asymmetric outcomes of asset purchases. We use this to derive a notion of "capacity constraints" for public interventions, whereby large (or "fast") asset sales by the government can distort welfare even if they improve risk sharing. That is, the government faces a trade-off between improving risk sharing and distorting inter-temporal smoothing, and the costs of inter-temporal distortions grow if the government tries to trade large quantities within a given period.

Despite that markets are complete and fully integrated, there is only limited spill-over of government trading and demand shocks to substitutable assets. For example, if the government purchases short-term debt, it reduces short rates with only limited effects on long rates. This is because limited liquidity introduces endogenous trading costs that deter investors from deviating from their existing portfolios. Hence, our model complements existing frameworks in which spill-overs are limited because of market segmentation and "preferred habitats" (Vayanos and Vila, 2021). We illustrate this insight in an application of our model to the yield curve.

Taken together, our results provide theoretical support for the idea that large-scale asset purchases affect market liquidity and trading efficiency outside of crises, and that otherwise expansionary policies may have unintended consequences. We further illustrate how government trading that breaks budget-balance is non-neutral even in liquid markets, confounding its impact on liquidity in financial markets with that on enhancing risk-bearing capacity. Although our model is not intended to offer an exhaustive analysis of monetary-fiscal policy, it is well-suited for analyzing how a government may use trading rules to improve liquidity and trading efficiency in financial markets.

To this end, we derive simple rules for government tax-and-trading schemes that optimally improve risk sharing and inter-temporal trading arrangements, assuming that the government maximizes a social objective that assigns Pareto weights to different market participants.[5] We find that the government can minimize distortions to risk sharing and inter-temporal smoothing by choosing its portfolio holdings to equalize weighted averages of government (or public) and private agent price impacts. When the government trades risk-free debt, these weights are Pareto weights *times* marginal-utility averages of investor asset positions and position elasticities.

Next, we use our model to provide some insights into the types of securities that governments may want to trade. This is relevant for understanding broader asset market interventions that have been used in recent years. For example, the U.S. government traded agency mortgage-backed securities during the Global Financial Crisis and corporate bonds during the COVID-19 crisis, while Japan trades equity Exchange-Traded Funds (ETFs) to affect stock prices. This leads to a number of important questions regarding the relatively desirability of intervening in different asset classes. We first provide a generalization of our optimal trading rules to arbitrary securities, and then establish a *liquidity pecking order*: a government that needs to buy assets should buy more liquid assets first, such as short-maturity U.S. Treasuries and equities, to minimize the purchase's impact on market liquidity. In contrast, when it needs to sell assets, it should sell less liquid assets first, such as long-maturity U.S. Treasuries, to foster overall liquidity.

We conclude by using a simple calibration exercise to assess the financial market consequences of the 2014-2017 Eurozone Quantitative Easing program, and to highlight the rich heterogeneity permitted by our framework. Our goal is not to provide an exhaustive assessment of quantitative easing policies, but rather to use our model to provide a theory-guided analysis of implications for market functioning and risk sharing among financial institutions. This is important because recent evidence across asset markets suggests that financial institutions' price elasticities of demand are much lower than implied in classical asset pricing models. We show that our model can rationalize these low elasticities when calibrated to portfolio characteristics, such as duration and demand elasticities, and price effects, such as the risk-free rate and yield response to the large-scale asset purchase program. Interpreting the effects of the program facts through the lens of our fully micro-founded model, we find that the program lowered sovereign debt yields with only a modest loss in trading efficiency. This is because low demand elasticities imply

[5] We recognize that this objective does not consider investors outside the model, such as households and firms that may be ultimately affected by such large-scale asset purchase and sale programs. Hence, we view this objective mainly as a convenient way of aggregating payoffs across agents in the model.

that portfolio positions changed only modestly in response to the program.

Related literature. We contribute to the literature studying the implications of public asset purchases for market liquidity. Often, quantitative easing and large-scale asset purchases are studied in the context of financial crises with fire sales (e.g., Davila and Korinek (2018)). A recurring theme in this literature is that a government can alleviate the downward spiral in asset prices by buying assets when prices are depressed because of forced sales by market participants. More recently, there is growing interest in the potential risks and unintended consequences of public liquidity provision. For example, Schmid, Liu, and Yaron (2021) shows that although government debt issuance can improve market liquidity, it can increase risk in the economy and raise firms' cost of capital. Li (2024) examines how quantitative easing can mitigate bank liquidity crises but makes treated banks vulnerable to fluctuations in the real economy. Wallen and Stein (2023) finds that heterogeneity in the demand elasticities among money market funds can amplify how Treasury yields and reverse repurchase rates respond to supply shocks when Treasury bills are scarce. Du, Hebert, and Li (2023) shows that the increase in the supply of U.S. treasuries can explain why interest rate swap-treasury spreads turned negative post-global financial crisis. We provide a conceptual framework for understanding how government asset purchases operate outside of financial crises.

We also contribute to the literature on the public provision of risk-free debt. To the best of our knowledge, we are the first to study how a government should trade in concentrated financial markets to foster risk sharing and liquidity, filling an important gap in our understanding of the optimal design of large-scale asset purchases and sales. Many papers emphasize that when markets are incomplete and agents are competitive, government-issued risk-free debt can complete markets (e.g., Angeletos (2002)) and provide a savings alternative (e.g., Bewley (1986)). More recently, Caramp and Singh (2023) study how safe asset provision can mitigate the risk of liquidity traps, while He, Krishnamurthy, and Milbradt (2019) study the self-fulfilling nature of safe asset issuance. Choi, Kirpalani, and Perez (2022) argue that the U.S. government's market power over safe assets has led to their undersupply despite these benefits.

More broadly, this literature has emphasized that trading frictions, such as limits to arbitrage (e.g., Gertler and Karadi (2013)), market segmentation (e.g., Droste, Gorodnichenko, and Ray (2021)), and asymmetric information (e.g., Wang (2023)), are necessary for large-scale asset purchase programs to affect asset prices and real outcomes. For instance, Vayanos and Vila (2021) illustrates how such purchases can impact different parts of the yield curve when investor demand is segmented across maturities. Different from these models, we consider an economy with complete markets to focus on how the public provision of risk-free debt interacts with private risk-management, and show how imperfect liquidity introduces a cost of buying large amounts of safe assets. Our analysis demonstrates how government trading can have real effects even when there are no exogenous barriers to trade, forced asset sales, or asymmetric information, but there are lost gains from trade purely because of illiquidity.

Our paper also relates to a literature that studies market concentration in financial markets, and bond markets in particular. Eisenschmidt, Ma, and Zhang (2022) examines how large dealers' market power impacts the transmission of monetary policy in European repo markets. Wang (2018) studies how monetary policy transmission is affected by the market power of financial intermediaries, while Huber (2023) and Wallen (2020) study dealer market power in tri-party repo and foreign exchange derivatives markets, respectively. We ask how government trading can affect private trading efficiency by ameliorating the dearth of risk sharing. Eisenbach and Phelan (2022) studies fire sales externalities while Kacperczyk, Nosal, and Sundaresan (2021) investigates the impact of large institutional investors on asset price informativeness. Doerr, Eren, and Malamud (2023) examines how strategic money market mutual funds interact with banks in T-bill and repo markets. Basak (1997) and Basak and Pavlova (2004) examine asset pricing with a monopolistic non-price-taking agent in an Arrow-Debreu economy, the later in the context of a firm whose market power erodes under time-consistent policies. Neuhann and Sockin (2024) uses a similar framework to ours to study capital misallocation in a production economy, while Neuhann, Sefidgaran, and Sockin (2022) ask how market power varies with the span of assets that investors can trade. Here, we study an endowment economy with complete markets and use it to analyze the effects of government trading.

2 Model

There are two dates, $t = \{1, 2\}$. Uncertainty is represented by a set of states of the world $\mathcal{Z} \equiv \{1, 2, \ldots Z\}$, one of which realizes at date 2. The probability of generic state $z \in \mathcal{Z}$ is

$\pi(z) \in (0,1)$, and all agents share common beliefs.

Demographics. There are two classes of agents: a continuum of competitive agents with mass m_f called the *competitive fringe* who takes prices as given, and a discrete number of *strategic agents* who are large relative to the economy and internalize their impact on prices in financial markets. The presence of a competitive fringe can represent, for instance, retail investors and smaller institutional investors. There is also a government that can buy or sell risk-free debt, but is constrained to balance its budget at each date.

There are N *types* of strategic agents, indexed by $i \in \{1, 2 \ldots, N\}$, where an agent's type determines her income process. Within each type, there exist $1/\mu$ *symmetric* agents who each has mass μ. For an individual strategic agent j of type i, μ determines how much she internalizes her *price impact* because it affects her size relative to the economy. In the aggregate, average μ proxies for *market concentration*, or the extent to which the same wealth and income is concentrated in the hands of a few investors. In what follows, we focus on equilibria in which strategic agents within each type follow symmetric strategies.

Preferences. Strategic agents share common preferences over consumption at both dates. These are represented by the utility index $u(c)$ that is $\mathcal{C}^2$, strictly increasing, strictly concave, homothetic, and satisfies the Inada condition. Marginal utility $u'(c)$ is further assumed to be strictly convex. Risk aversion captures the notion that even large financial institutions can exhibit limited risk-bearing capacity under a variety of frictions, such as capital and risk management constraints. The fringe has quasi-linear preferences: linear in consumption at date 1 and risk-averse at date 2. Its date-2 utility function, $u_f(c)$, satisfies the same properties as that of strategic agents. Although a price-taking fringe is essential for our results, quasi-linearity of its preferences is not.[6]

Income and Consumption. The fringe receives initial wealth w_f and state-contingent endowment $y_f(z) > 0$. A strategic agent j of type i receives initial endowment μw_i at date 1, and state-contingent endowment $\mu y_i(z) > 0$ in state z. The total initial endowment and state-contingent income of agents of type i are consequently also w_i and $y_i(z)$, respectively, and the *aggregate endowment* of all strategic agents is $Y(z) = \sum_i y_i(z)$. These income processes can be interpreted in multiple ways. One interpretation is that they represent the operational cash flow exposures of institutional investors. Another is that they repre-

[6]Earlier versions considered a fringe with the same risk-averse preferences at date 0 as at date 1. Although this yields a richer price impact function than with quasi-linearity because of wealth effects, it complicates the analysis without qualitatively adding to our insights on the role of government trading.

sent the payoffs of asset portfolios that were in place before the government intervenes. Risk sharing needs then could represent the outcome of shocks to the expected payoffs of these portfolios. In the context of insurance companies and pension funds, these could reflect not only differences in existing asset exposures, but also in net cash flows from premiums less payouts to insurees or defined benefit pensioners.

Aggregate resource constraints are as follows. Let $c_{1,j,i}$ and $c_{2,j,i}(z)$ denote consumption of agent j of type i at date 1 and in state z, respectively, and similarly with c_{1f} and $c_{2,f}$ for the fringe. Aggregating within types gives $c_{1,i} = \sum_{j=1}^{1/\mu} \mu c_{1,j,i}$ and $c_{2,i}(z) = \sum_{j=1}^{1/\mu} \mu c_{2,j,i}(z)$. The aggregate resource constraints are

$$\begin{aligned}\sum_{i=1}^{N} c_{1,i} + m_f c_{2,f} &= \sum_{i=1}^{N} w_i + w_f, \\ \sum_{i=1}^{N} c_{2,i}(z) + m_f c_{2,f}(z) &= Y(z) + m_f y_f(z).\end{aligned}$$

Financial Markets. Financial markets are complete and open at date 1. The traded assets are the full set of Arrow securities; i.e., there are Z securities such that security z pays one unit of the numeraire in state z and zero otherwise. We show below that equilibrium allocations are invariant to the precise security menu, holding fixed the asset span. As such, it is without loss of generality to focus on trading in Arrow securities only.

Let $a_{j,i}(z) \in \mathbb{R}$ denote the position of agent j of type i in claim z, where $a_{j,i}(z) < 0$ denotes a sale. Aggregating within and across types yields $a_i(z) \equiv \sum_{j=1}^{1/\mu} \mu a_{j,i}(z)$ and $A(z) \equiv \sum_{i=1}^{N} a_i(z)$. The fringe's and the government's positions in security z are $a_f(z)$ and $a_G(z)$, respectively. Market clearing in the market for claim z requires:

$$A(z) + a_G(z) + m_f a_f(z) = 0. \tag{1}$$

Finally, define $\mathbf{A}$ to be the $(N+2) \times Z$ matrix summarizing portfolios choices of all agents and the government. The equilibrium price function of asset z is denoted $Q(\mathbf{A}, z)$. In contrast, the *perceived pricing functional* used by agent j of type i to forecast her influence on the price of security z is $\tilde{Q}_{i,j}(\mathbf{A}, z)$.

Government. The government can either buy or sell Arrow assets at date 1 subject to budget balance at each date. It maintains budget balance through uniform lump sum transfers τ_1 and $\tau_2(z)$ at dates 1 and 2 to all agents that can be positive or negative. This imposes the budget constraints

$$\left(N+m_f\right)\tau_1+\sum_{z\in\mathcal{Z}}\tilde{Q}_G(\mathbf{A},z)a_G\left(z\right)=0, \tag{2}$$

$$\left(N+m_f\right)\tau_2\left(z\right)+a_G\left(z\right)=0. \tag{3}$$

While we maintain this general formulation of taxes and transfers throughout, a natural interpretation of these tax-and-trade schemes is that the government exchanges long-term financial assets (i.e., the Arrow securities) for cash or reserves held by large financial institutions. As will become clear, these exchanges can have equilibrium consequences because exchanging cash for financial assets may induce financial institutions to alter their assset portfolios even when their budget set is unchanged.

Our complete-markets setting with Arrow securities allows us to flexibly capture a variety of financial market interventions. For example, if we want to impose the restriction that the government trades only risk-free debt, we need only impose that $a_G(z)=a_g$. That is, if the government demands a_g units of risk-free debt, this equivalent to buying a_g units of each Arrow security $z\in\mathcal{Z}$, and similarly if it sells a_g units. Because markets are complete, this is equivalent to the government instead trading only risk-free debt in a risk-free debt market. However, we also permit other asset-specific policies.

Decision Problems and Equilibrium Concept. The government is a Stackelberg leader and sets its tax and trading policies first. Conditional on these policies, we search for a *Cournot-Walras* equilibrium in which the competitive fringe takes asset prices as given and strategic agents place limit orders while taking into account their price impact. This equilibrium concept differs from the Equilibrium-in-demand-schedules approach in the tradition of Kyle (1989). An advantage of our setting is its much greater tractability, which allows us to study complete markets and incorporate rich hetereogeneity across investors and trading needs. This is critical for our purposes because we can rule out that government policy operates by completing markets, and allows us to discuss distributional consequences of market interventions. Neuhann and Sockin (2024) provides a more detailed comparison of the two equilibrium concepts.

A *strategy* $\sigma_{j,i}$ for strategic agent j of type i consists of asset positions and consumption, $\sigma_{j,i} = \{\{a_{j,i}(z)\}_{z\in\mathcal{Z}}, c_{1,j,i}, c_{2,j,i}\}$. The decision problem is

$$
\begin{aligned}
U_{j,i} = \max_{\sigma_{j,i}} \quad & u\left(c_{1,j,i}\right) + \sum_{z\in\mathcal{Z}} \pi\left(z\right) u\left(c_{2,j,i}\left(z\right)\right) \\
\text{s.t.} \quad & \mu c_{1,j,i} = \mu w_i - \mu\tau_1 - \sum_{z\in\mathcal{Z}} \tilde{Q}_{i,j}(\mathbf{A}, z)\mu a_{j,i}\left(z\right), \\
& \mu c_{2,j,i}\left(z\right) = \mu y_i\left(z\right) + \mu a_{j,i}\left(z\right) - \mu\tau_2\left(z\right).
\end{aligned} \tag{4}
$$

We define preferences and controls in this manner recognizing that the consumption of strategic agent j of type i is actually $\mu c_{1,j,i}$ and $\mu c_{2,j,i}\left(z\right)$ at dates 1 and 2, respectively, and similarly with optimal asset holdings, $\mu a_{j,i}\left(z\right)$. Given homothetic utility, however, optimal policies are invariant to defining a strategic agent's preferences over $\mu c_{t,j,i}$.

A *strategy* σ_f for the competitive fringe consists of asset positions and consumption, $\sigma_f = \{\{a_f(z)\}_{z\in\mathcal{Z}}, c_{1,f}, c_{2,f}\}$. Because it takes prices as given, its perceived pricing function satisfies $\tilde{Q}_f(\mathbf{A}, z) = \tilde{Q}_f(z)$. The fringe's decision problem is

$$
\begin{aligned}
U_f = \max_{\sigma_f} \quad & c_{1f} + \sum_{z} \pi(z) u(c_{2,f}(z)) \\
\text{s.t.} \quad & c_{1f} = w_f - \tau_1 - \sum_{z} \tilde{Q}_f(z) a_f(z), \\
& c_{2,f}\left(z\right) = y_f(z) - \tau_2\left(z\right) + a_f(z).
\end{aligned} \tag{5}
$$

This allows us to define our equilibrium concept as follows.

Definition 1 (Cournot-Walras Equilibrium) *Fixing a strategy of the government, a Cournot-Walras equilibrium consists of a strategy $\sigma_{j,i}$ for each strategic agent, a strategy σ_f for the competitive fringe, and pricing functions $Q(\mathbf{A}, z)$ for all $z \in \mathcal{Z}$ such that:*

1. *Fringe optimization: σ_f solves decision problem (5) given $\{\tilde{Q}_f(z)\}_{z\in\mathcal{Z}}$*

2. *Strategic agent optimization: For each agent j of type i, $\sigma_{j,i}$ solves decision problem (4) given (i) other agents' strategies $\{\sigma_{-j,i}, \sigma_f\}$ and perceived pricing functions $\{\tilde{Q}_{j,i}(\mathbf{A}, z)\}_{z\in\mathcal{Z}}$.*

3. *Market-clearing: Each market clears with zero excess demand according to (1).*

4. *Consistency: all agents have rational expectations, which requires for strategic agents that $\tilde{Q}_{j,i}(\mathbf{A}, z) = Q(\mathbf{A}, z)$ for all i, j and z.*

The competitive fringe intermediates strategic interaction in our model. Although a strategic agent takes the asset positions of other strategic agents as given, he does internalize how his own demand impacts equilibrium asset prices by altering the marginal utility of the fringe. Through this channel, how one strategic agent type trades *indirectly* affects how another strategic agent type trades by altering the prices (and price impact) that agent type faces.

3 Optimal Portfolios and Properties of Equilibrium

We now characterize fundamental properties of equilibrium. In models of strategic trading, a crucial step is characterizing the pricing functional that determines an investors' equilibrium influence on prices. In our setting, this is simplified by the fact that the price-taking competitive fringe optimally aligns its marginal utility in a given state with the associated Arrow security price. Asset prices are consequently pinned down by the fringe's consumption process. Holding other large agents' portfolios fixed, each large agent can then infer her price impact from how much the fringe's marginal utility will move when she demands more or less of a given security. Because a strategic agent's influence scales with her mass, μ, her individual price impact does as well. This is shown in Lemma 1.

Lemma 1 (Prices and Price Impact) *The price of the Arrow security referencing state z is*

$$Q(\mathbf{A}, z) = q(z) \equiv \pi(z) u_f' \left(c_{2,f}(z)\right). \tag{6}$$

The price impact of strategic agent i satisfies

$$\frac{\partial Q_{j,i}(\mathbf{A}, z)}{\partial a_i(z)} = \frac{\mu}{m_f} q'(z) \qquad \text{where} \qquad q'(z) \equiv \frac{\partial q(z)}{\partial A(z)} = -\pi(z) u_f'' \left(c_{2,f}(z)\right) > 0, \tag{7}$$

and price impact $q'(z)$ is increasing and convex in strategic agent demand. The law of one price holds, and equilibrium consumption allocations are invariant to the presence of a risk-free or redundant assets. With increasing, concave utility, price impact is increasing in the price level.

In addition to determining asset prices and price impact, Lemma 1 has two additional important implications. The first is that the law of one price holds, so that consumption allocations are invariant to the asset span. As such, we do not need to explicitly model a risk-free asset. The second is that price impact vanishes in the limit as $\mu \to 0$. As such, our model nests perfect competition as a benchmark. Since the equilibrium is efficient under perfect competition, deviations from the perfect-competition limit allow us to distill welfare consequences.

Government Policies as Asset Endowments. A useful way of understanding the effects of government trading is to interpret public trading as endowing private investors with an inventory of assets that is either in line with privately desired trading or must be undone through financial markets. In particular, if the government's portfolio is $\{a_g(z)\}$, budget balance requires that

$$\tau_1 = \frac{1}{N+m_f} \sum_{z \in \mathcal{Z}} q(z)\, a_G(z) \qquad \text{and} \qquad \tau_2(z) = -\frac{1}{N+m_f} a_g(z).$$

These are state-contingent transfers that endow the agents with a certain "ex-ante" state-contingent consumption process, but is budget neutral in the sense that the government is paying market prices. If markets were perfectly liquid, such changes in "asset endowments" would be entirely neutral as private investors could always undo them by taking the reverse position in asset markets.

With imperfect liquidity, however, unwinding government positions is costly, and so it is never optimal to fully undo the effects of government trading. This allows the government to influence asset prices and private portfolios using budget-balanced interventions in complete markets. To capture the net effects of public and private trading on an investor's state-contingent income, we define *normalized* asset positions as

$$\hat{a}_{j,i}(z) = a_{j,i}(z) + \frac{a_G(z)}{N+m_f}. \tag{8}$$

Consumption and asset prices are then determined by normalized asset positions,

$$c_{1i} = w_i - \sum_{z \in \mathcal{Z}} q(z)\, \hat{a}_{j,i}(z) \qquad \text{and} \qquad c_{2i}(z) = y_i(z) + \hat{a}_i(z).$$

where

$$q(z) = \pi(z)\, u_f' \left(y_f(z) - \frac{1}{m_f} \sum_{i=1}^{N} \hat{a}_{j,i}(z) \right).$$

Optimal Portfolios and Inelastic Trading. We can now formally state the conditions for the optimal portfolio choice of each strategic agent. Marginal valuations can be summarized by the *state price* of strategic agent j of type i, $\Lambda_{j,i}(z) \equiv \frac{\pi(z)u'\left(c_{2,j,i}(z)\right)}{u'\left(c_{1,j,i}\right)}$, and depends only on normalized asset positions.

Lemma 2 (Optimal Portfolio) *At an optimum, asset positions $\{a_i(z)\}$ satisfy the first-order necessary conditions*

$$\Lambda_{j,i}(z) - q(z) = \frac{\mu}{m_f} q'(z) \underbrace{\left(\hat{a}_i(z) - a_G(z)/(N + m_f)\right)}_{=a_i(z)}. \tag{9}$$

The left-hand side of equation (9) shows marginal valuations and asset prices, which are optimally equal to each other in competitive markets. The right-hand side captures the distortions induced by price impact, which is the motive to ration quantities to preserve favorable terms of trade. The distortion scales with position size $a_i(z)$ because this affects the infra-marginal importance of the terms of trade. It is also defined over the *gross position* $a_i(z)$, and not the normalized asset positions $\hat{a}_i(z)$. This is because price changes affect the terms of trade on all assets that need to be reallocated, irrespective of the final normalized position.

We next establish that there is too little trade in the Cournot-Walras Equilibrium compared to the competitive benchmark. This is because a strategic agent optimally rations her asset purchases and sales to improve her terms-of-trade. Because all strategic agents do this, in aggregate there is too little trade and too few gains from trade are realized. This is summarized in Lemma 3.

Lemma 3 (Insufficient Trade) *There is less trade asset-by-asset in the Cournot-Walras than in the competitive equilibrium.*

We next show that investors with price impact trade inelastically, in the sense that asset quantities respond sluggishly to shocks that need to be accommodated through financial markets. For concreteness, we do so in context of wealth-neutral demand shocks, which are shocks that increase diversifiable income risk but do not alter total resources available in the economy (state-by-state). While such shocks are neutral under perfect competition, they are not when investors face price impact.

Definition 2 *A wealth-neutral demand shock is a reshuffling of large investor endowments $y_i(z)$ that leaves unchanged the total endowment in each state $\sum_{i=1}^{N} y_i(z)$ and the present-value of each endowment under prevailing market prices $\sum_{z\in\mathcal{Z}} q(z)\, y_i(z)$.*

Lemma 4 (Inelastic Trading) *In the Cournot-Walras equilibrium where large investors have price impact, a wealth-neutral demand shock:*

(i) alters asset prices and consumption allocations even though it would have been entirely neutral in the perfect competition limit where $\mu \to 0$.

(ii) worsens the efficiency of risk sharing, as measured by state price dispersion, if it raises the endowment of sellers and lowers that of buyers in all asset markets.

In concentrated financial markets, investors do not efficiently absorb demand shocks because price impact induces them to trade too little. As such, consumption allocations and asset prices are more responsive to demand shocks than under perfect competition, asset quantities are less responsive, and the efficiency of risk sharing falls when it increases gains from trade (i.e., distributes more resources to sellers and less to buyers). The response of asset prices to demand shocks in this setting is driven by both a direct and an indirect effect. The direct effect is that the imperfect reallocation of consumption in a specific state of the world because of market concentration alters the state price for that state of large investors, and consequently how they trade with the fringe. This alters the fringe's consumption in that state and consequently the associated Arrow price. The indirect effect is that a change in the price of one asset alters the wealth of all large investors, which alters all their state prices and consequently their demands for other assets. This spillover effect gives rise to cross-asset elasticities from demand shocks.

Measuring equilibrium risk sharing distortions. Since our theory is focused on risk management, we want to measure the efficiency of risk sharing in a model-consistent manner. Inefficient risk sharing leads to lost gains from trade, which are differences in state prices across investors. Hence, we can measure the inefficiency of risk sharing in a state z as the cross-investor *dispersion* in state prices:

$$\omega(z) = \frac{1}{N}\sum_{i=1}^{N}\left(\Lambda_i(z) - \frac{1}{N}\sum_{j=1}^{N}\Lambda_j(z)\right)^2. \tag{10}$$

While state prices are unobservable, portfolio positions and prices are not. Using the first-order condition (9), we can substitute out state prices using observable measures to arrive at the risk-sharing wedge,

$$\omega(z) = \left(\frac{\mu}{m_f} q'(z)\right)^2 \frac{1}{N} \sum_{i=1}^{N} \left(a_i(z) - \frac{1}{N} \sum_j a_j(z)\right)^2 \tag{11}$$

As such, trading efficiency is directly linked to two channels: price impact, which deters the realization of gains from trade, and dispersion in gross quantities, which are linked to the underlying gains from trade.

4 Positive Effects of Government Trading

In the previous section, we characterized the basic properties of strategic agents' portfolios and the extent to which they share risks under price impact. In this section, we derive positive and normative implications of government trading in asset markets. For realism and starkness, we restrict the government to trading risk-free debt. This is a natural benchmark intervention because it is not only simple, but practically relevant. It also means that government interventions are *neutral* across investors: the government does not take a distorted position in any single security, and all taxes and transfers are symmetric across all investors. Despite this neutrality, we show that government trading can still affect the degree of risk sharing that occurs in equilibrium. In the last subsection, we relax the assumption of government budget-balance to study the additional effect of shifting aggregate resources across dates from unfunded asset purchases.

4.1 Benchmark: Ricardian Equivalence under Perfect Competition

We first show that, absent price impact, government trading is completely neutral with respect to consumption allocations and asset prices.

Benchmark 1 (Ricardian Equivalence) *In the limit with perfect competition ($\mu \to 0$), government purchases and sales do not affect consumption allocations or asset prices. Moreover, risk sharing is perfect and there are no lost gains from trade for any government policy.*

The reason for this neutrality is that all costs and profits of government trading are passed on and rebated to investors, respectively. Since trade is frictionless absent price impacts, investors can therefore always undo any undesirable effects of government policies by adjusting their gross positions in financial markets, leaving net positions unchanged.

4.2 Prices and Liquidity

We now turn to the model with price impact. As the Ricardian benchmark in the previous section illustrates, it is not obvious that the government can affect equilibrium asset prices, and therefore price impact. We now show that it does. To build intuition, we use the portfolio condition from Lemma 2 to derive a simple asset pricing equation. Summing the optimality condition across strategic agents and imposing market-clearing shows that prices are satisfy a distorted consumption-based equation,

$$q(z) = \frac{1}{N}\sum_i \Lambda_i(z) + \frac{\mu}{m_f} q'(z) m_f a_f(z) + \frac{\mu}{m_f} q'(z) a_g. \tag{12}$$

The first term on the right-hand side of equation (12) is familiar from consumption-based asset pricing in which asset prices reflect the average marginal valuation of investors, as determined by the marginal rate of substitution. The remaining two terms reflect that the average wedge between asset prices and state prices is related to the net demand absorbed by the remaining investors in the market, i.e., the competitive fringe and the government. The third term specifically suggests that the direct effect of government trading is standard: prices fall when the government sells, and rise when it buys. We show this is indeed the case in the following proposition, and derive implications for liquidity as well. The risk-free rate is defined as the inverse sum of asset prices,

$$r_f = \left(\sum_{z\in\mathcal{Z}} q(z)\right)^{-1}. \tag{13}$$

Proposition 1 (Price and Liquidity Effects of Government Trading) *In the model with price impact ($\mu > 0$), budget-balanced public tax and trading schemes affect asset prices as follows:*

(i) public purchases of risk-free bonds raise prices and price impact of all assets, and lower the risk-free rate.

(ii) public sales of risk-free bonds lower prices and price impact of all assets, and raise the risk-free rate.

Proposition 1 represents our first main result, which is that the government can improve market liquidity by reducing price impact by budget-neutral trading schemes that would be entirely neutral under perfect competition. This is because price impact deters private investors from fully undoing government purchases via financial markets, leading to equilibrium consequences of budget-neutral policies. Since asset purchases raise prices, "expansionary" policies which serve to lower risk-free rates reduce liquidity by raising price impact. In our model without financial constraint, the intuition is that high asset prices indicate scarcity and high marginal values of consumption, which make it costly to reallocate consumption on the margin. In the next section, we show that this mechanism leads to worse risk sharing in response to public debt purchases.

4.3 Effects on Risk Sharing

Given that the government can affect liquidity, it is natural investigate whether it can improve trading efficiency. Liquidity improvements alone are not enough to guarantee this because risk sharing distortions are the product of price impact and trading quantities. In particular, (11) shows that lost gains from trade are given by

$$\omega(z) = \left(\frac{\mu}{m_f} q'(z)\right)^2 \frac{1}{N} \sum_{i=1}^{N} \left(a_i(z) - \frac{1}{N} \sum_j a_j(z)\right)^2$$

This expression highlights that the overall effects on risk sharing are driven by both price impact and gross trading volumes. As such, public interventions which lower price impact may still worsen risk sharing if they sufficiently raise trading volumes.

To theoretically characterize the net effects of government interventions on risk sharing absent any other considerations, we first restrict attention to income processes under which the only gains from trade because of risk sharing. In such economies, all gains from trade are orthogonal to the payoffs of risk-free assets. This ensures that government trades in risk-free debt do not directly affect risk sharing. We also restrict attention to an economy populated almost exclusively by strategic agents, which we call the *strategic limit*. This allows us to isolate the effects of government purchases in the cleanest possible setting. To ensure that price impact remains well-defined in the limit, we consider the joint limit where $\mu \to 0$ and μ/m_f converges to a nontrivial constant.[7]

[7] Neuhann and Sockin (2024) provides a formal analysis of this particular limit economy.

Definition 3 (Pure Risk Sharing Economy) *A pure risk sharing economy is one where all strategic agents are ex-ante symmetric but ex-post heterogeneous. As such, they face identical decision problems up to a relabeling of the states.*

Definition 4 (Strategic Limit) *The strategic limit is the limit of a sequence of economies in which* $\mu, m_f \to 0$ *and* $\mu/m_f \to \kappa$ *for some constant* $\kappa > 0$.

We begin by summarizing the benchmark allocation that obtains in concentrated markets absent government interventions.

Benchmark 2 (Asset Prices without the Government) *In the strategic limit of the pure risk sharing economy, all Arrow asset prices are inflated above their competitive equilibrium counterparts (i.e, prices are higher than when* $\mu = 0$.*)*

The proof can be adapted from Neuhann and Sockin (2024) to the case where large agents have endowments instead of production technologies. The underlying mechanism is that, for any asset, supply curves for are always more elastic than demand curves. The reason is that sellers *choose* to sell precisely because they are rich when the asset pays off, and thus have relatively flat marginal utility. Hence sellers always ration supply more than buyers ration demand, and all asset prices are too high relative to the efficient benchmark.

We then have our second main result, which is that public purchases of risk-free debt reduce the efficiency of risk sharing. Since purchases of risk-free debt also lower interest rates, our model predicts that expansionary policies brought about by quantitative interventions are associated with inefficient risk taking by financial institutions.

Proposition 2 (Effects of Government Trading on Risk Sharing) *In the strategic limit of a pure risk sharing economy, risk sharing distortions (as measured by the dispersion of state prices* $\omega(z)$*) are increasing in government asset purchases* a_g *for all states* z*. Conversely, government asset sales lead to a decline in risk sharing distortions.*

This result is striking because the payoffs of risk-free assets are orthogonal to gains from trade due to risk sharing, and all wealth effects are neutralized by taxes and subsidies. As such, our mechanism operates *only* through improved liquidity and by crowding in efficient asset supply. In particular, because sellers are the "marginal distorters" in the economy, selling risk-free debt can improve risk sharing allowing rationed buyers to obtain more insurance. As a result, state price dispersion declines and risk sharing improves. The converse argument can be made for government asset purchases.

While Proposition 2 relied on the strategic limited to obtain analytical results for relatively general income processes, this is not necessary. In particular, the following example considers an analytically tractable pure risk sharing economy with two strategic types, two states, and a "large" competitive fringe. This setting allows us to derive clear analytical expressions that highlight private distortions to risk sharing and the government's role in ameliorating them.

Example 1 *There are two types of strategic agents, $i \in \{1, 2\}$ and two states $z \in \{1, 2\}$ that are equally likely. Endowments satisfy $y_1(1) = 2\bar{y}$ and $y_1(2) = 0$, and $y_2(1) = 0$ and $y_2(2) = 2\bar{y}$. All agents have an initial wealth w. The fringe receives $\frac{\bar{y}}{w}$ in every state. There are Arrow assets for states 1 and 2, both of which will have equilibrium price q^* by symmetry. Preferences are of the CRRA type and taxes and transfers are such that $\tau_1 = \frac{2}{2+m_f} q^* a_g$ and $\tau_2(z) = -\frac{1}{2+m_f} a_g$.*

Since the two strategic agent types are symmetric, we can search for an equilibrium where Type 1 agents sell $a_s < 0$ units of the claim to state 1 and buy $a_b > 0$ units of the claim to state 2. Type 2 agents take the reverse positions. Define $\hat{a}_s = a_s + \frac{a_g}{2+m_f}$ and $\hat{a}_b = a_b + \frac{a_g}{2+m_f}$. The claim in each state has a price q^* based on the fringe's marginal utility, $q^* = u'\left(\frac{\bar{y}}{w} - \frac{1}{m_f}(\hat{a}_b + \hat{a}_s)\right)$. and price impact is q'^*. Moreover, strategic agents net expenditures on assets at date 1 are $q^*(\hat{a}_b + \hat{a}_s)$. As such, asset positions satisfy

$$\text{Seller optimality:} \qquad \frac{\frac{1}{2}u'(2\bar{y} + \hat{a}_s)}{u'(w - q^*(\hat{a}_b + \hat{a}_s))} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{a_g}{2+m_f}\right), \tag{14}$$

$$\text{Buyer optimality:} \qquad \frac{\frac{1}{2}u'(\hat{a}_b)}{u'(w - q^*(\hat{a}_b + \hat{a}_s))} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{a_g}{2+m_f}\right). \tag{15}$$

With perfect competition (i.e., $\mu = 0$), $a_b = -a_s = \bar{y}$, and government intervention in financial markets has no real effects. With market concentration, in contrast, sellers sell fewer claims ($a_s > -\bar{y}$) while buyers buy fewer claims ($a_b < \bar{y}$) because of price impact. As a result, Type 1 agents are over-exposed to state 2 risk, while Type 2 agents are over-exposed to state 1 risk.

If the government sells a small amount of claims, or $a_g = \epsilon < 0$, then government sales reduce the market concentration wedge for the seller type, $\frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{\epsilon}{2+m_f}\right)$, and increase it for the buyer type, $\frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{\epsilon}{2+m_f}\right)$. The direct effect from the first-order conditions is to induce the seller type to sell more claims and the buyer type to buy fewer claims. The direct effect decreases *the demand that the fringe must absorb $\hat{a}_b + \hat{a}_s$, reducing the claim price q^* and consequently price impact q'^* (the indirect effect). The indirect effect that lowers the claim price and price impact, in turn, mitigates the reduction in purchases by the buyer type. On net, this improves risk sharing according to $\omega(z)$ by raising $\frac{\frac{1}{2}u'(2\bar{y}+\hat{a}_s)}{u'(w-q^*(\hat{a}_b+\hat{a}_s))}$ more than it lowers (or raises) $\frac{\frac{1}{2}u'(\hat{a}_b)}{u'(w-q^*(\hat{a}_b+\hat{a}_s))}$ because the seller's supply curve is more elastic in each market.*

4.4 Effects on Intertemporal Trade

Beyond risk sharing, the other motive for trade is inter-temporal smoothing across investors with different income duration. We now ask how government trading affects this margin. As under pure risk sharing, price impact and inelastic trading lead to inefficient inter-temporal smoothing whenever market participants face income streams of different duration. The key difference to the case of risk sharing is that government purchases of risk-free debt are in *same* direction as at least some agents in the economy. This leads to distributional effects that differ from those under risk sharing, and is particularly relevant when analyzing duration mismatch among pension funds and insurance companies.

For simplicity, we focus on the following transparent setting. There two types of strategic agents, $i \in \{1, 2\}$. Endowments satisfy $y_1 = 2y$, and $y_2 = 0$. Type 1 agents have initial wealth 0 while Type 2 agents have initial wealth $2y$. The fringe receives 1 at date 2. There is a risk-free bond with equilibrium price q^*. Preferences are of the CRRA type and taxes and transfers are such that $\tau_1 = \frac{1}{2+m_f} q^* a_g$ and $\tau_2(z) = -\frac{1}{2+m_f} a_g$.

We search for an equilibrium where Type 1 agent sells $a_s < 0$ units of risk-free debt and Type 2 agents buy $a_b > 0$ units of risk-free debt. Define $\hat{a}_s = a_s + \frac{a_g}{2+m_f}$ and $\hat{a}_b = a_b + \frac{a_g}{2+m_f}$. Risk-free debt has a price q^* based on the fringe's marginal utility, $q^* = u'\left(1 - \frac{1}{m_f}(\hat{a}_b + \hat{a}_s)\right)$. and price impact is q'^*. Moreover, strategic agents net expenditures on assets at date 1 are $q^* \hat{a}_s$ and $q^* \hat{a}_b$, respectively. As such, asset positions satisfy

$$\text{Seller optimality:} \qquad \frac{u'(2y + \hat{a}_s)}{u'(-q^* \hat{a}_s)} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{a_g}{2 + m_f}\right), \tag{16}$$

$$\text{Buyer optimality:} \qquad \frac{u'(\hat{a}_b)}{u'(2y - q^* \hat{a}_b)} = q^* + \frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{a_g}{2 + m_f}\right). \tag{17}$$

With perfect competition (i.e., $\mu = 0$), $a_b = -a_s = y$, $q^* = u'(1) = 1$ with CRRA preferences, and government intervention in financial markets has no real effects. With market concentration, in contrast, Type 1 agents sell fewer claims ($a_s > -y$) while Type 2 agents buy fewer claims ($a_b < y$) because of price impact. As a result, the assets of Type 1 agents have too long a duration, and the assets of Type 2 agents have too short a duration.

If the government sells a small amount of risk-free debt, or $a_g = \epsilon < 0$, then the direct effect reduce the market concentration wedge for the seller type (Type 1 agents), $\frac{\mu}{m_f} q'^* \left(\hat{a}_s - \frac{\epsilon}{2+m_f} \right)$, and increase it for the buyer type (Type 2 agents), $\frac{\mu}{m_f} q'^* \left(\hat{a}_b - \frac{\epsilon}{2+m_f} \right)$. As a result, Type 1 agents sell more claims while Type 2 agents buy fewer claims. This reduces the demand the fringe must absorb $\hat{a}_b + \hat{a}_s$, reducing the risk-free debt price q^* and consequently price impact q'^* (the indirect effect), which mitigates the reduction in purchases by the buyer type. This improves inter-temporal smoothing by raising Type 1's state price $\frac{u'(2y+\hat{a}_s)}{u'(-q^*\hat{a}_s)}$ more than it lowers (or raises) Type 2's $\frac{u'(\hat{a}_b)}{u'(2y-q^*\hat{a}_b)}$.

4.5 Endogenous Capacity Constraints

The previous sections showed that the government can improve the efficiency of trade by crowding in the supply of rationed assets, no matter the underlying gains from trade. We now argue that the model also gives rise to a notion of endogenous "capacity" constraints that may determine limits on the appropriate size of interventions. To see this, recall that the effect of price impact on optimal portfolios is determined by the product of trade volumes and price impact. Absent the government, market clearing forces some investors to take short positions if other take long positions. For sufficiently large government interventions, however, *all* private investors may take a long or short position, sharply raising gross private trading volumes. Observe that equations (14)-(17) imply that gross trading volumes in the pure risk sharing or the inter-temporal smoothing economy are

$$Vol = |\hat{a}_s - \frac{a_g}{2+m_f}| + |\hat{a}_b - \frac{a_g}{2+m_f}|$$

where $\hat{a}_s < 0 < \hat{a}_s$ for a_g sufficiently close to zero. This leads to the following observation.

Corollary 1 (Capacity Constraints) *Government sales reduce gross trade volumes if $a_g < (2+m_f)\hat{a}_b$ but raise gross trade volumes if $a_g \geq (2+m_f)\hat{a}_b$.*

Since trading efficiency is related to the product of price impact and gross volumes, large interventions may therefore lower trading efficiency even if they reduce price impact.

4.6 General Gains from Trade: Risk Sharing and Inter-temporal Trade

We now integrate our results on risk sharing and inter-temporal smoothing. In particular, we consider an economy in which there are gains from trade stemming from both risk sharing and inter-temporal smoothing needs. The next result establishes that government asset sales, on the margin, can improve risk sharing in the economy, while asset purchases worsen it if the fringe has sufficiently limited risk-bearing capacity.[8] However, the the government still faces a trade-off even when it can improve risk sharing using asset sales. In particular, the cost of distorting inter-temporal smoothing may locally reduce welfare even if risk sharing improves. As such, the government may face "capacity constraints" beyond which financial markets cannot appropriately accommodate government interventions, even if the government trades in the appropriate direction.

Since endowments can be interpreted as pre-determined asset holdings, this result has the practical implication that sufficiently "fast" asset sales may be sub-optimal relative to smaller interventions. This allows our model to speak to events such as the Gilt market crash in the fall of 2022, where relatively sudden quantitative tightening by the Bank of England *revealed* that large institutional investors were poorly hedged ex-ante (for example, because central banks had previously maintained large positive balance sheets), and this distorted inter-temporal trade among investors with different duration.

Proposition 3 (General Gains from Trade) *Let $\underline{\gamma}$ be the competitive fringe's minimum coefficient of absolute risk aversion across all asset markets z in the absence of government intervention. Government asset purchases cannot achieve the competitive outcome. In addition, for each z:*

(i) if the government buys a small amount risk-free bonds and $\frac{\underline{\gamma}}{m_f}$ is sufficiently large, this lowers risk sharing efficiency in the economy by raising $\omega(z)$. Large asset purchases worsen risk sharing further by driving $\omega(z)$ to its autarky value.

(ii) if the government sells a small amount risk-free bonds and $\frac{\underline{\gamma}}{m_f}$ is sufficiently large, this improves risk sharing efficiency in the economy by lowering $\omega(z)$. Large asset sales have diminishing returns by driving $\omega(z)$ for each z to an asymptotic, but positive lower bound.

(iii) Since asset sales may distort inter-temporal trade, it may not be welfare-optimal to drive

[8] While this condition is sufficient and intuitive, it is not necessary: we find that the same mechanism holds numerically under quite general conditions.

the risk sharing wedge to its lower bound. In particular, smaller interventions may deliver higher welfare than large interventions.

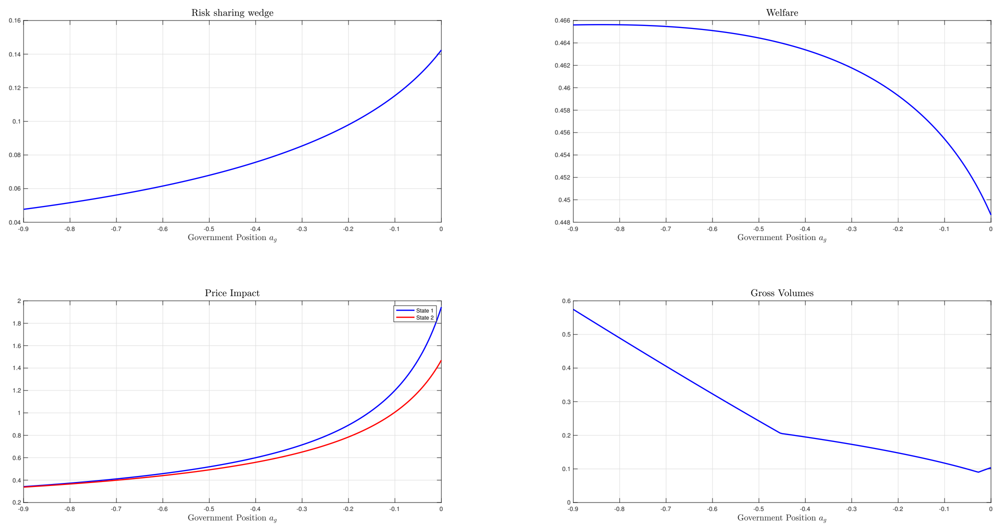

Figure 1: Effects of government trading given inter-temporal and risk sharing gains from trade.
Remarks. We compute equilibrium allocations in the strategic limit of the two-state, two-type economy discussed in Example 1, with the modification that types are also allowed to differ in their initial wealth. The ratio of market concentration to fringe mass is $\frac{\mu}{m_f} = 1$. Average income in every state is $\bar{y} = 1$. The within-state income dispersion that determines risk sharing needs is $\Delta = 0.3$. Type 1's initial wealth is $w_1 = 1$. There are inter-temporal gains from trade because Type 2's initial income is higher, $w_2 = 2.5$. The risk sharing wedge is the average of the state-contingent wedges, $\frac{1}{2}\sum_{z=1}^{2}\omega(z)$. Liquidity as measured by price impact is plotted separately for each Arrow security. Gross volumes are the sum of all gross Arrow security positions. Welfare is utilitarian welfare as in Equation (20).

Figure 1 illustrates our results by plotting equilibrium outcomes as a function of the government's position a_G. We study the strategic limit of the simple two-state, two-type from Example 1, enriched to allow for inter-temporal gains from trade stemming from differences in initial wealth. The top left panel shows that the risk sharing wedge, which measures the inefficiency of risk sharing, falls as the government sells more risk-free debt. The bottom left panel shows that the underlying mechanism is in part driven by improving liquidity (falling price impact) for all assets. The top right panel shows that utilitarian welfare is hump-shaped: increasing for relatively small interventions, but falling for sufficiently large interventions. The bottom right panel reveals the source of this non-linearity: for sufficiently large interventions, all investors trade "against" the government to partially undo the policy. Since this sharply raises gross volumes, the price impact friction grows in importance, reducing the efficiency of the allocation. This shows the importance of endogenous "capacity constraints" (Corollary 1).

4.7 On the Effects of the Funding Structure

As is well-known in the public finance literature, the effects of public policies often depend on how government expenditures are funded. So far, we have assumed that government operations are fully funded by non-distortionary taxes and transfers on market participants. A natural interpretation of these tax-and-trade schemes is that the government exchanges long-term assets for cash or reserves held by large financial institutions. However, in practice these interventions may be "unfunded" from the perspective of market participants. For example, government interventions may be funded by current or future taxation on agents outside of the model. To understand how the funding structure affects the equilibrium outcomes of government interventions, we now weaken the assumption of budget balance in every period. We then show that unfunded asset purchases have the additional effect of shifting aggregate resources across dates.

Suppose the government has an endowment y_{1g} at date 1 and y_{2g} at date 2, and its objective is to trade until it equates its consumption at both dates

$$y_{1g} - \sum_{z \in \mathcal{Z}} q(z)\, a_g = y_{2g} + a_g, \tag{18}$$

from which follows, defining $\Delta y_g = y_{1g} - y_{2g}$ that

$$a_g = \frac{\Delta y_g}{1 + \sum_{z \in \mathcal{Z}} q(z)}. \tag{19}$$

This has the interpretation of the government wanting to inelastically smooth its expenditures at each date by issuing or purchasing risk-free debt. By varying Δy_g (the endowment mismatch), this fixed rule will provide an exogenous source of demand (or supply) to financial markets at date 1. We focus on the strategic limit from Definition 4 in which $\mu, m_f \to 0$ but $\frac{\mu}{m_f} \to \kappa$ and shut down all taxes and transfers. To examine the role of large-scale asset purchase programs like quantitative easing, we assume $y_{1g} < y_{2g}$ so that $a_g = \bar{a}_g > 0$ and the government buys risk-free debt. The complementary case when $y_{1g} < y_{2g}$, which captures the more quotidian behavior of governments to issue debt to finance expenditures, has analogous results. We then have the following proposition.

Proposition 4 (Unfunded Large-Scale Asset Purchases) *Suppose Δy_g increases and the government demands more risk-free debt. Then, under both perfect and imperfect competition, all asset prices $q(z)$ rise and all agents consume more at date 1 and less at date 2.*

Proposition 4 shows that unfunded government large-scale asset purchases break Ricardian Equivalence. They effectively reallocate resources among strategic agents from date 2 to date 1 by crowding-out investment in risk-free bonds. This induces agents to consume more at date 1. In a more general setting, such a crowding-out of investment in risk-free bonds can crowd-in investment into alternative investment opportunities, such as riskier equities or corporate debt. For instance, Joyce, Liu, and Tonks (2017) show that the Bank of England's quantitative easing program during the Global Financial Crisis shifted the investments of U.K. insurance companies and pension funds from Gilts toward corporate bonds. Our analysis also clarifies the distinct mechanisms through which government trading influences market liquidity, which operates regardless of the funding scheme, and how it affects the market's risk-bearing capacity, which requires that trading be unfunded.

5 Limited Spill-overs in Integrated Markets

A central question in the evaluation of large-scale asset purchases is how such interventions spill over to other asset markets. Hence, we use our model to examine whether government purchases of one asset are transmitted to other assets toward which investors might be willing to substitute. In our model, such spill-overs are possible because markets are integrated and investors face no exogenous portfolio constraints. To illustrate this mechanism in a particularly salient setting, we study dynamic extension of our model with multiple maturities of risk-free debt. This allows us to study how government interventions in one maturity propagate along the yield curve, which is a central consideration for many real-world asset purchase programs.

Specifically, consider a variant of our model with three dates $t \in \{1, 2, 3\}$ and Z_t states of the world at date t, with a generic state indexed by z_t. Investors have time-separable preferences at all three dates and do not discount the future. They trade Arrow securities in complete financial markets which open date 0 and close at date 1.

The government can influence the short- or long end of the yield curve by, respectively, buying a quantity a_{gt} of the set of Arrow securities referencing date $t \in \{2,3\}$. The associated risk-free rates are $r_{ft} = \left(\sum_{z_{tk} \in \mathcal{Z}_t} q\left(z_{tk}\right)\right)^{-1}$. Budget balance requires transfers $\tau_t = -\frac{1}{N+m_f} a_{gt}$ for $t \in \{2,3\}$ and $\tau_1 = \frac{1}{N+m_f}\left(\sum_{z_{2k} \in \mathcal{Z}_2} q\left(z_{2k}\right) a_{g2} + \sum_{z_{3k} \in \mathcal{Z}_3} q\left(z_{3k}\right) a_{g3}\right)$. As in the baseline model, these policies are entirely neutral if markets are perfectly competitive. With imperfect competition, instead, specific interventions do affect market prices, particularly for the assets that are directly purchased.

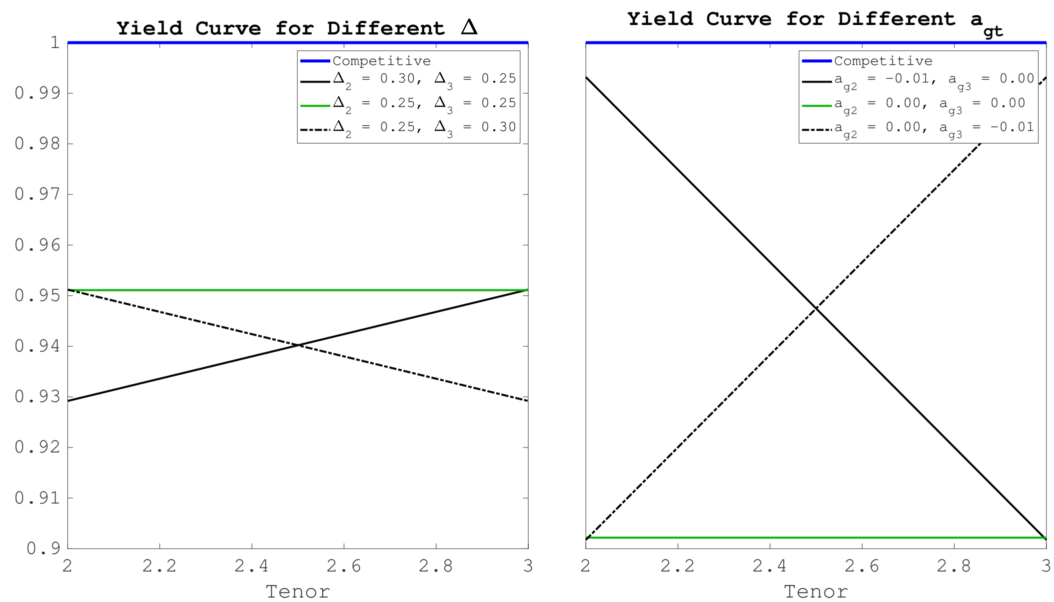

Figure 2: Term structure as a function of gains from trade Δ_t (left panel) and government asset sales a_G right panel. We use the following setting. There are two types strategic agents, $i \in \{1,2\}$. At dates 2 and 3, there are two possible states $z \in \{1,2\}$ with $\pi(z) = \frac{1}{2}$. Strategic agents face pure idiosyncratic risk: $y_i(it) = \bar{y} + \Delta_t$ and $y_i(-it) = \bar{y} - \Delta_t$. The fringe receives $\bar{y}$ in every state. All agents have log preferences and initial endowments of w_i and w_f for strategic agents and the fringe, respectively. The parameters are $m_f = 0.01, \mu = 0.10, \bar{y} = 2, w_f = 0$, and $w_1 = w_2 = 1$. The baseline values are $a_{g2} = a_{g3} = 0$ for the left panel and $\Delta = 0.25$ for the right panel

Figure 2 shows this effect by plotting the term structure of interest rates at tenors 2 and 3 using a simple numerical example. The left panel shows yield curves for our preferred measure of gains from trade, namely dispersion in endowments Δ_t. A shock to the short rate engineered by higher income dispersion at date 2 (i.e., higher Δ_2) steepens the yield curve by lowering the date 2 risk-free rate and raising the date 3 risk-free rate, while a shock to the long rate through a higher Δ_3 has the opposite effect of inverting the yield curve. Hence, there is limited pass-through of shocks that are, in principle, perfectly diversifiable and would be entirely neutral in competitive markets. The right panel shows yield curves for different levels of government asset sales a_{gt}. Similar to the left panel, government demand shocks are only partially passed through: asset sales at date 2 (i.e., $a_{g2} < 0$) raise the date 2 risk-free rate and modestly lower the date 3, while asset sales at date 3 (i.e., $a_{g3} < 0$) steepen the yield curve by raising the date 3 risk-free rate and modestly lowering that at date 2. In either case, pass-through is imperfect and the intervention primarily affects the targeted assets.

This limited pass-through is striking because all demand shocks and government policies could, in principle, be reversed through financial markets. That pass-through is imperfect therefore reveals that the model operates similar to a preferred habitat framework in which pass-through is limited by market segmentation (e.g., Vayanos and Vila (2021)). Our theory instead relies on endogenous trading costs rather habitats to generate behavior which looks "as if" investors are constrained in moving away from their ideal portfolios. This leads to a complementary mechanism by which quantity-based interventions such as large-scale asset purchase programs can impact interest rates at a specific tenor of the yield curve even though markets are completed and fully integrated.

6 Optimal Liquidity Trading Rules

The previous sections characterized positive consequences of public asset market interventions, taking as given government trading rules. To evaluate the net benefit of interventions, we now formally examine how a government would optimally buy or issue risk-free debt to maximize social welfare. The government assigns strategic agents of type i a Pareto weight of θ_i and to the fringe a Pareto weight of θ_f, such that $\sum_{i=1}^{N} \theta_i + \theta_f = 1$, which we summarize in a vector $\Theta = [\theta_1, ..., \theta_N, \theta_f]$, and consequently maximizes

$$W(\Theta, a_g) = \sum_{i=1}^{N} \theta_i \left(u(c_{1,j,i}) + \sum_{z \in \mathcal{Z}} \pi(z) u(c_{2,j,i}) \right) + \theta_f m_f \left(\sum_{z \in \mathcal{Z}} \pi(z) u(c_{2,f}) - c_{1,f} \right). \tag{20}$$

The welfare objective is a convenient way of aggregating payoffs across agents in the model that illustrates how policy interventions can both raise overall trading efficiency but also induce redistributive effects. We allow the government to assign different Pareto weights across market participants to reflect that the government may prioritize some subset of market participants (i.e., systemically important financial institutions).

The government is a Stackelberg leader in that it declares a budget-balanced tax and trading policy and internalizes all investors' reactions to this policy. We then have the following analytical characterization of the optimal policy rule for risk-free debt.[9]

Proposition 5 (Optimal Government Debt) *The government's optimal holding of risk-free debt satisfies the necessary condition:*

$$\sum_{z\in\mathcal{Z}} w_p(z)\frac{\mu}{m_f}q'(z) - w_g(z)\frac{dq(z)}{da_g} = 0\ (< if a_g = -\infty, > if a_g = \infty), \tag{21}$$

where $w_p(z) = \sum_{i=1}^N \theta_i u'(c_{1,i})\, a_i(z)\frac{d\hat{a}_i(z)}{da_g}$ *and* $w_g(z) = \sum_{i=1}^N \left(\theta_i u'(c_{1,i}) - \theta_f\right)\hat{a}_i(z)$ *are the weights on private and public price impact, respectively. This condition is trivially satisfied at the competitive equilibrium where price impact is zero.*

Proposition 5 reveals that the government chooses its risk-free debt position based on a weighted average across states, with weights $w_p(z)$ and $w_g(z)$, of the difference between private $\frac{\mu}{m_f}q'(z)$ and public (or the government's) price impact $\frac{dq(z)}{da_g}$, respectively. A strategic investor's price impact multiplied by her position $\frac{\mu}{m_f}q'(z)\,a_i(z)$ represents the wedge between her marginal valuation and marginal cost of consumption in a given state. This wedge is zero when there is perfect risk sharing in the competitive benchmark. The government's price impact is the total derivative of the price with respect to its trading position because it internalizes that it shifts private agents' demand in equilibrium. It is the government's ability to impose taxes and its internalization of its impact on prices (i.e., the total vs partial derivative) that introduces a role for policy. The government, as the Stackelberg leader, recognizes it can influence other large agents' asset demands, and not just the fringe's.

The government chooses its risk-free debt position until, on average across states, it counteracts large investors' private price impact. In markets in which $w_p(z)\frac{\mu}{m_f}q'(z) - w_g(z)\frac{dq(z)}{da_g}$ is positive, the government wants to sell debt if it has a higher weighted price impact than private agents to provide buyers relief with lower prices and price impact. In contrast, when $w_p(z)\frac{\mu}{m_f}q'(z) - w_g(z)\frac{dq(z)}{da_g}$ is negative, the government wants to buy debt to help sellers if it has higher weighted price impact. Importantly, a higher Pareto weight for strategic agent i corresponds to a higher weight in both weights $w_p(z)$ and w_g

[9]Alternatively, we could search for government positions, a_g, that constitute Pareto improvements, i.e., $W(\Theta, a_g) \geq W(\Theta, 0)$ under every set of Pareto weights θ in the $N+1$-simplex.

in all markets, so that the government aims at aligning its public impact more with that agent's price impact.

From the optimality condition (21), it is apparent that social welfare can be non-monotonic in the government's holdings of risk-free debt. This is because there may be an interior choice of risk-free debt that balances the redistribution and erosion of rents among strategic agents, given their Pareto weights, with the improvement in risk sharing when the government sells debt, and the reverse when the government buys risk-free debt. The erosion of rents is more pronounced the more concentrated are financial markets (i.e., higher μ), which can lead to an interior choice of optimal risk-free debt.

6.1 Targeted Interventions and a Liquidity Pecking Order

The role of public liquidity in concentrated financial markets is not specific to risk-free debt. In this subsection, we consider more-targeted interventions to general asset classes. These interventions are of theoretical interest and practical relevance: addition to sovereign bonds, many governments, including the US and EU, intervened in corporate bond markets in 2020 during the COVID crisis, and Japanese government is now the largest holder of Japanese equities.

Trading rule for arbitrary securities We can adapt our optimal trading rule to allow the government to trade a richer set of linearly-independent securities. In particular, for a security with state-contingent payoff vector $X(z)$ and price q_x, its position $a_{\tilde{g}x}$ should satisfy at an interior solution

$$\sum_{z \in \mathcal{Z}} X\left(z\right) \left(w_p\left(z\right) \frac{\mu}{m_f} q'\left(z\right) - w_g\left(z\right) \frac{dq\left(z\right)}{da_{gx}} \right) = 0, \tag{22}$$

where $w_p\left(z\right)$ and $w_p\left(z\right)$ are given in Proposition 5. The government continues to align weighted public and private price impact, but achieves more targeted interventions based on distortions to the specific asset market.

There are two immediate implications of allowing the government to trade in asset markets other than risk-free debt. First, is that it can target assets in which agents with higher Pareto weights trade to improve their welfare. These assets have payoffs $X\left(z\right)$ that correspond to higher weights $w_p\left(z\right)$ and $w_g\left(z\right)$, all else equal. Second, that it can trade in relatively more liquid securities (i.e., those with higher expected returns, $\sum_z X(z)/q_x$) or in relatively more illiquid securities (i.e., those with lower expected returns) than risk-free debt. Expected returns (which correct for differences in payoffs) reflect liquidity because (high) low returns are indicative that such securities are in (plentiful) dear supply for risk-sharing and have (positive) negative risk premia. Trading in relatively low return securities, such as derivatives, has a profound effect on financial markets because of their high price impact. In contrast, assets with high returns that are highly liquid, such as equities, can be traded in large quantities without drastically affecting the risk management practices of large market participants.

Pecking order. Our second insight suggests a natural pecking-order to asset interventions. If the government sells assets to improve risk sharing and the risk management practices of market participants, it should prioritize trading in more illiquid assets that have a larger impact on market liquidity. In contrast, if buys assets to support specific agents (based on the Pareto weights), then it should prioritize trading in more liquid securities to minimize its adverse impact on market liquidity. Such a prescription, selling in illiquid and buying in liquid assets, can provide guidance to governments on how to conduct LSAPs, and potentially rationalize the Bank of Japan's historical preference for buying liquid securities like equity Exchange-traded Funds to provide stimulus to its economy.

7 Calibration to the Eurozone

Our theoretical results show that government trading can have rich and nuanced effects on market outcomes. To study how government asset purchases might impact equilibrium trading arrangements, asset prices, and welfare in practice, we calibrate our model to Eurozone data from 2014-2017 based on Koijen, Koulischer, Nguyen, and Yogo (2021). We emphasize that our goal is *not* to provide an exhaustive assessment of quantitative easing policies on the macroeconomy (for example, by accounting for its spillovers to household and firms), but rather to use our model to provide a theory-guided analysis of implications for market functioning and risk sharing among financial institutions. The calibration also illustrates an important advantage of our framework, which is that we can allow for rich cross-sectional heterogeneity and flexible payoff structures even in a model with strategic trading.

The 2014 intervention is a useful case study for our aims because the European Central Bank conducted large-scale asset purchases outside of a crisis period, which allows us to examine how such purchases impact market liquidity absent forced asset sales. A novelty of our calibration is that we directly target cross-sectional characteristics of the portfolio holdings and demand elasticities of different institutional investors. In what follows, we consider date 1 to represent one year and date 2 to represent ten years; as such, risk-free debt in our model is akin to a ten-year government bond and the risk-free rate the ten-year rate on government bonds.

To calibrate our model to Koijen, Koulischer, Nguyen, and Yogo (2021), we assume that there are three groups of strategic agents: 1) insurance companies and pension funds (ICPFs) that have long duration portfolios; 2) banks and corporations that have short duration portfolios; and 3) mutual funds and hedge funds that have portfolios of intermediate duration. All strategic agents have constant relative risk aversion (CRRA) preferences with risk aversion γ, as does the competitive fringe at date 2. To focus on risk sharing among these institutional investors, we examine the strategic limit from Definition 4.

To flexibly capture the distinct portfolios and trading motives of these different agent types, we specify initial wealth and endowments as follows. At date 2, there are two possible states $z \in \{1, 2\}$ with $\pi(z) = \frac{1}{2}$. Strategic agents of type $i \in \{1, 2\}$ that represent pension funds and insurance companies receive initial wealth $(1 - k_1)\,\bar{y}$ and an endowment at date 2, $y_i(i) = k_1\bar{y}\,(1 + \Delta)$ and $y_i(-i) = k_1\bar{y}\,(1 - \Delta)$. That is, in every state one of the two types has high income and the other has low income. Similarly, strategic agents of type $i \in \{3, 4\}$ that represent banks and corporations receive initial wealth $(1 - k_2)\,\bar{y}$ and an endowment at date 2, $y_i(i - 2) = k_2\bar{y}\,(1 + \Delta)$ and $y_i(5 - i) = k_2\bar{y}\,(1 - \Delta)$. Strategic agents of type $i = 5$ that represent hedge funds and mutual funds receive initial wealth $(1 - k_3)\,\bar{y}$ and a certain endowment at date 2 of $k_3\bar{y}$. The competitive fringe receives $\bar{y}$ at both dates.

We set $\bar{y} = 10$, $\kappa = 1$, and calibrate the remaining parameters $\{\gamma, k_1, k_2, k_3, \Delta, a_g\}$ as follows. With time-separable utility the coefficient of relative risk aversion γ also determines the elasticity of inter-temporal substitution. Given that inter-temporal smoothing is the key source of gains from trade among the institutions we study, we target γ to match the risk-free rate r_f with the Euro area 10 Years Government Bencnhmark Bond

Yield in March 2014 of 2.8%.[10]. We target k_1 , k_2, and k_3 to match the durations of government bond holdings of IPCFs, banks and corporations, and mutual funds from Table 14 of Koijen, Koulischer, Nguyen, and Yogo (2021). We weight the durations of each group across vulnerable and non-vulnerable countries by the size of their holdings to arrive at values of 8.94 years, 4.62 years, and 6.92 years, respectively.[11] We measure duration D in our model using Macaulay's Duration for strategic agent i portfolio, D_i, based on the fraction of present-value consumption derived at each date

$$D_i = \frac{c_{1i}}{w_i + \sum_{z=1}^{2} q\left(z\right) y_i\left(z\right) - \tau_1 - \frac{1}{r_f}\tau_2} + 10\frac{\sum_{z=1}^{2} q\left(z\right) c_{2i}\left(z\right)}{w_i + \sum_{z=1}^{2} q\left(z\right) y_i\left(z\right) - \tau_1 - \frac{1}{r_f}\tau_2}. \tag{23}$$

We target Δ to match the mean demand elasticity for risk-free bonds of ICPFs from Table 13 of Koijen, Koulischer, Nguyen, and Yogo (2021) of -4.04. Defining $\hat{a}_1(rf) = \min_{\{z\in\{1,2\}} a_i\left(z\right)$ to be agent i's holding of risk-free bonds, we can calculate this demand elasticity as $-\frac{d\log|\hat{a}_1(rf)|}{d\log\left(1/r_f\right)}$. Finally, we target the initial size of government trading a_g such that a 26% purchase of outstanding government bonds (the effective size of the Eurozone's asset purchases from 2014-2017) reduces the ten-year yield by 65 bp based on the calculations of Koijen, Koulischer, Nguyen, and Yogo (2021).

Table 1 reports the parameters that we recover from estimating our model using the simulated method-of-moments approach, and Table 2 compares the calibrated moments that we target in our model with their empirical counterparts. It is worth emphasizing the ambition of our exercise in that it is very difficult to match not only asset pricing moments (i.e., the risk-free rate and yield changes), but also portfolio characteristics (i.e., duration and demand elasticities) that are typically ignored in models of strategic trading. Of note is that our calibrated model estimates very realistic demand elasticities for strategic agents compared to the perfect competition benchmark in which elasticities are (locally) infinite. Although we target only the demand elasticity for risk-free debt of ICPFs, those of banks/corporates and mutual/hedge funds in our model are also relatively low at 6.80 and 23.83 (compared to 2.08 and 2.93 in Koijen, Koulischer, Nguyen, and Yogo (2021)), respectively, with mutual and hedge funds sensibly being the

[10]See https://data.ecb.europa.eu/data/datasets/FM/FM.M.U2.EUR.4F.BB.U2_10Y.YLD.

[11]Specifically, ICPFs have an average duration of (1284 * 9.8 + 493 * 6.7) / (1284 + 493) = 8.94 years. Banks have an average duration of (1346 * 5.0 + 963 * 4.1)/ (1346 + 963) = 4.62 years. Mutual funds, which include hedge funds, have an average duration of (895 * 7.6 + 333 * 5.1) / (895 + 333) = 6.92 years.

most elastic. In addition, price impact in risk-free bonds, i.e., $\kappa \frac{q'}{q}$ for bond price q, is 0.30. While the calibrated coefficient of relative risk aversion is low, we view this as indicating the inter-temporal smoothing is particular important for the institutions we study, so that our preferred interpretation is in the context of an elasticity of inter-temporal substitution.

Parameter	Interpretation	Value
$\bar{y}$	Average Endowment	10.000
κ	Relative Size of Strategic Agents	1.000
k_1	Fraction of Bank/Corporate Endowment at Date 2	0.2973
k_2	Fraction of ICPF Endowment at Date 2	0.6198
k_3	Fraction of Mutual/Hedge Fund Endowment at Date 2	0.6703
Δ	Distributional Endowment Risk (% of $\bar{y}$)	0.0109
γ, γ_f	Agent Risk Aversion	0.3117
a_g	Government Initial Asset Position	-0.4258

Table 1: Parameter choices for the baseline calibration.

Moment	Data	Model
Ten-year Risk-free Rate	1.29%	1.43%
Bank / Corporate Duration	4.62	4.21
ICPF Duration	8.94	6.21
Mutual/Hedge Fund Duration	6.92	6.54
ICPF Demand Elasticity	-4.04	-3.86
Asset Purchase Yield Response	65bp	64bp

Table 2: Model vs empirical moments for the parameters given in Table 1.

Figure 3 illustrates our novel risk-taking channel according to which strategic agents retain too much diversifiable risk when asset prices are high and interest rates are low. We simulate the equilibrium for different values of the government's position a_g. The vertical line plots the calibrated "initial position" of the government. Starting from this point, asset purchases thus represent a rightward move along the x-axis. An advantage of our no-arbitrage framework with complete markets is that we can rewrite Arrow security positions in terms of more interpretable assets. In the left panel, we plot the normalized positions of insurance companies and pension funds (ICPFs) in terms of a risk-free bond $\hat{a}_1(rf)$ and a swap that pays 1 in state 2 and -1 in state 1, $\hat{a}_1(swap)$. In the middle panel, we plot the risk-free rate, and in the right panel, we plot our risk-sharing efficiency measure $\omega(z)$ (which is the same across both states in this exercise).

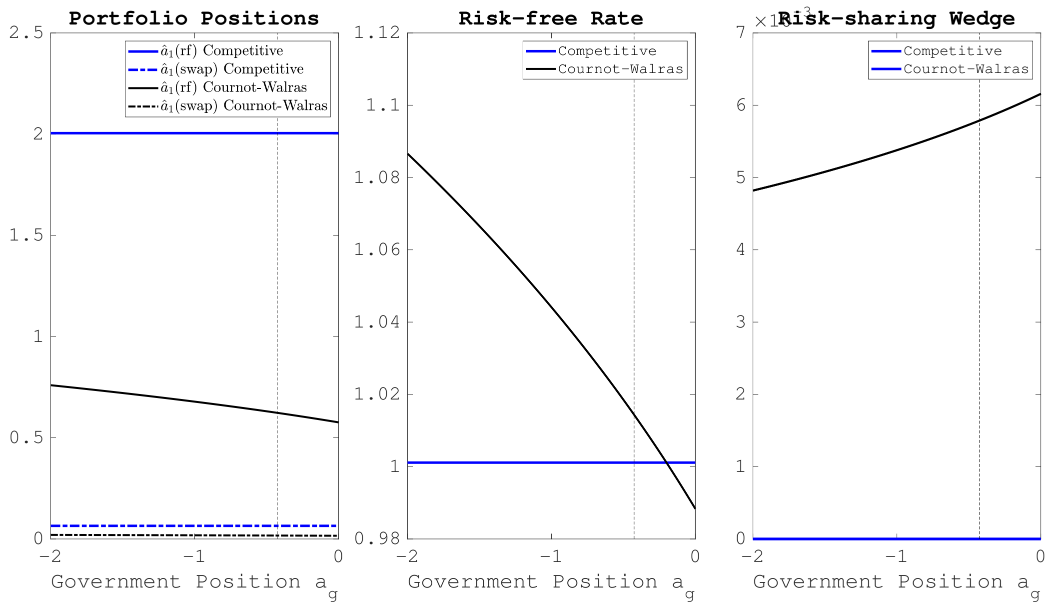

Figure 3: Agent 1's Holdings (Left Panel), Risk-free Rate (Middle Panel), and Risk-sharing Efficiency $\omega(z)$ (Right Panel) Across Government Positions for the parameters in Table 1

When interest rates are low, ICPFs trade too little of their diversifiable risk. In the competitive equilibrium, they should trade 2.005 shares of risk-free debt and 0.0648 shares of the swap. Instead, they trade only 0.6232 shares of risk-free debt and 0.0169 shares of the swap, voluntarily over-exposing themselves to their own state and mismatched duration to extract surplus. As the government sells more assets and interest rates rise, their positions in risk-free debt and the swap increase to 0.7210 and 0.0191 shares, respectively when $a_g = -1.25$, leading agents to realize more gains from trade in financial markets. However, because the demand elasticities of agents are very low, government trading has only limited impact in improving risk sharing. The right panel reveals that risk sharing improves as the government sells more assets and reduces price impact, although it cannot achieve the level of risk sharing in the competitive equilibrium.

Our analysis consequently cautions that examining only (even large) changes in yields is insufficient for evaluating the transmission of large-scale asset purchases to investor portfolios and their implications for welfare.

We plot welfare according to our utilitarian objective (20) in Figure 4. Although welfare is always below the competitive benchmark (0.84% lower consumption-equivalent welfare in the baseline specification), it is decreasing in public debt purchases. As the government buys debt, the risk-free rate falls and price impact rises. This induces ICPFs to buy less bonds and to take a smaller position in the swap that shares risk. However, because ICPFs, banks, and mutual funds all have very inelastic demands, the 26% large-scale asset purchase by the European Central Banks government bonds over 2014-2017 mostly impacted prices rather than allocations. As a result, it modestly reduced consumption-equivalent welfare by -0.012%. Our analysis consequently cautions that examining only (even large) changes in yields is insufficient for evaluating the transmission of large-scale asset purchases to investor portfolios and their implications for welfare.

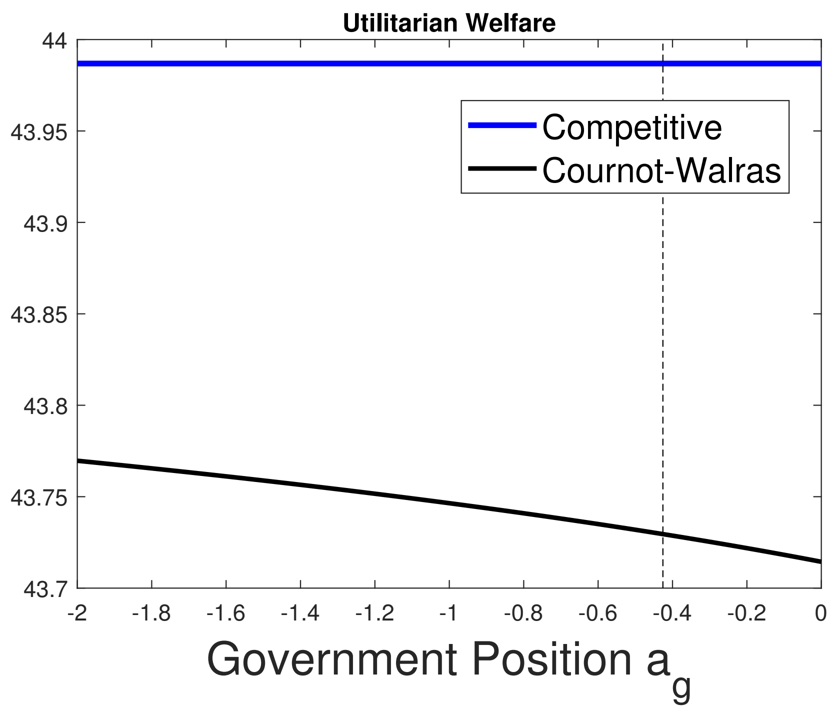

Figure 4: Welfare Across Government Positions for the parameters in Table 1

8 Conclusion

We provide a novel perspective of how the public provision of liquidity impacts risk sharing among investors with price impact. In otherwise frictionless financial markets, government asset sales can improve liquidity by attenuating financial market concentration. In contrast, when a government buys assets and lowers interest rates, investors instead intensify rent-seeking at the expense of risk management. Our results can help explain why, after a long period of government asset purchases and low interest rates, many large institutional investors are now highly exposed to diversifiable risks. Our results on optimal policy deliver insights into the optimal management of public portfolios and central bank balance sheets, and highlight that market liquidity introduces a natural pecking-order to how governments should intervene in asset markets.

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A Proofs of Propositions

A.1 Proof of Lemma 1:

As a preliminary, suppose we have some arbitrary asset span indexed by the $|\mathcal{Z}| \times |\mathcal{Z}|$ matrix X that is of full rank. In the special case of Arrow-Debreu assets, $X = I_{|\mathcal{Z}|}$, i.e., the identity matrix of rank $|\mathcal{Z}|$. Let x_k index the k^{th} row vector of X, and $x_k(z)$ be the dividend asset k pays in state z.

If the competitive fringe trades assets with asset span X, it is immediate from the first-order conditions of the competitive fringe's optimization problem that the vector of asset prices $\vec{q}_X$ satisfies:

$$\vec{q}_X = X\vec{\Lambda}_f, \tag{24}$$

where $\vec{\Lambda}_f$ is vector of the fringe's state prices. Since the quasi-linear competitive fringe now maximizes $u_f\left(y_f(z) - \sum_{k=1}^{|\mathcal{Z}|} x(z)\, x_k(z)\, A_{x_k}(z)\right) + \sum_{k=1}^{|\mathcal{Z}|} x(z)\, q_{x_k} A_{x_k}(z)$, where $A_{x_k}(z)$ is the total demand for asset k of the strategic agents, the price impact function can be summarized by the matrix Γ:

$$\Gamma = XUX', \tag{25}$$

where U is the diagonal matrix with diagonal entries $-\frac{\mu}{m_f}\pi(z)u_f''\left(c_{2,f}(z)\right)$.

Step 1: The Problem of the Fringe:

From the first-order condition for $a_f(z)$ from the competitive fringe's problem (5), we can recover the pricing equation of the Arrow-Debreu claim to security z:

$$\tilde{q}(z) = \pi(z)u_f'\left(c_{2,f}(z)\right) = \Lambda_f(z),$$

where $\Lambda_f(z)$ is the competitive fringe's state price. Since $c_{2,f}(z) = y_f(z) - \tau_2(z) + a_f(z)$, imposing the market-clearing condition, (1), reveals:

$$\tilde{q}(z) = \pi(z)u_f'\left(y_f(z) - \tau_2(z) - \frac{1}{m_f}\left(A(z) + a_G(z)\right)\right).$$

In equilibrium, this must be the realized price of the claim, $Q(\mathbf{A}, z)$. As this price is a function of state variables from the perspective of the fringe, we designate the realized price more concisely as:

$$q\left(z\right)=Q(\mathbf{A},z).$$

Let $\vec{q}$ be the vector of Arrow asset prices.

Step 2: Equilibrium Price Impact:

Because agents of type i take the demands of other agents (even within their type) as given, $u_f\left(z\right)$ is twice continuously differentiable, and each agent's position size scales by its mass μ, we can derive each agent's perceived price impact:

$$\frac{\partial\tilde{Q}_{j,i}(\mathbf{A},z)}{\partial a_i\left(z\right)}=-\frac{\mu}{m_f}\pi(z)u_f''\left(c_{2,f}(z)\right)=-\frac{\mu}{m_f}\frac{\partial q(z)}{\partial A\left(z\right)},$$

which also implies that price impact is symmetric across all strategic agents. Defining $q'\left(z\right)=\frac{\partial q(z)}{\partial A(z)}$ yields the expression in the statement of the proposition.

Finally, we recognize price impact $q'\left(z\right)$ is convex because of the convex marginal utility of the fringe. It is straightforward to see:

$$q''\left(z\right)=\frac{\mu}{m_f}\pi(z)u_f'''\left(c_{2,f}(z)\right)>0,$$

$$q'''\left(z\right)=-\left(\frac{\mu}{m_f}\right)^2\pi(z)u_f''''\left(c_{2,f}(z)\right)>0.$$

Price impact is consequently convex in the net demand of strategic agents. In addition, because $-u_f''\left(c_{2,f}(z)\right)$ is (weakly) decreasing in $c_{2,f}(z)$ with increasing, concave utility, it follows prices and price impact are both increasing in the net demand of strategic agents and the government $A\left(z\right)$. Because $c_{2,f}(z)$ is a sufficient statistic for both price impact and the price level, it follows price impact is increasing in the price level.

Step 3: Strategic Agent Demand:

Consider the optimization problem of strategic agent j of type i, (4). We attach the Lagrange multiplier φ_i to the budget constraint. The first-order necessary conditions for

$c_{i,j,1}$ and $\{a_{i,j}(z)\}_{z\in\mathcal{Z}}$ are given by:

$$c_{1,j,i} \;:\; u'(c_{1,j,i}) - \varphi_{j,i} \leq 0 \;\; (= \; if \; c_{1,j,i} > 0), \tag{26}$$

$$a_{j,i}(z) \;:\; -\pi(z)\, u_2^{i\prime}(c_{2,j,i}(z)) + \varphi_{j,i}\left(\tilde{Q}_{j,i}(\mathbf{A}, z) + \frac{\partial \tilde{Q}_{j,i}(\mathbf{A}, z)}{\partial a_{j,i}(z)} a_{i,j}(z)\right) = 0. \tag{27}$$

The above represents the first-order necessary conditions for agent $i's$ problem. Because $u(\cdot)$ satisfies the Inada condition, $c_{1,j,i} > 0$ and (26) binds with equality.

Because strategic agent i has rational expectations, her perceived price impact must coincide with her actual price impact from (7). Consequently, equation (27) reduces to:

$$a_{j,i}(z) : \Lambda_{j,i}(z) = q(z) + \frac{\mu}{m_f} q'(z) a_{j,i}(z) \,\forall\, z \in \mathcal{Z}. \tag{28}$$

Step 4: The Law of One Price:

It is immediate from equation (24) because $\vec{q} = \vec{\Lambda}_f$:

$$\vec{q}_X = X\vec{q}. \tag{29}$$

The Law of One Price consequently holds if we introduce redundant assets into the complete markets economy.

Step 5: Invariance:

Let us conjecture that consumption allocations are unchanged if strategic agents and the fringe instead trade with asset span X. Because the fringe's consumption is unchanged, its state prices $\vec{\Lambda}_f$ and consequently Arrow prices $\vec{q}$ are unchanged.

Notice next we can stack the first-order conditions for strategic agent i with asset span $I_{|\mathcal{Z}|}$ from equation (28) as:

$$\vec{\Lambda}_i = \vec{\Lambda}_f + U\vec{a}_i, \tag{30}$$

where $\vec{\Lambda}_i$ are the stacked state prices of agent i, $\vec{a}_i$ is the vector of her asset positions, and we have substituted for Arrow-Debreu prices $\vec{q}$ with $\vec{\Lambda}_f$.

Let $\vec{a_{i,x}}$ be the vector of asset positions of agent i when she instead trades with asset span X. Imposing invariance of the consumption allocations of strategic agent i requires:

$$\vec{a_i} = X'\vec{a_{i,x}}. \tag{31}$$

Substituting with equation (31), we can manipulate equation (30) to arrive at:

$$X\vec{\Lambda}_i = X\vec{\Lambda}_f + XUX'\vec{a_{i,x}} = X\vec{\Lambda}_f + \Gamma\vec{a_{i,x}}, \tag{32}$$

where we have also substituted with equation (25). This is the identical stacked first-order conditions if strategic agent instead traded asset span X.

Both strategic agents and the competitive fringe therefore choose the same state-specific asset exposures under both asset spans. Finally, because the Law of One Price holds, the cost of each agent's portfolio is the same under both asset spans. We conclude that consumption allocations are invariant to the complete markets asset span.[12]

A.2 Proof of Lemma 2:

The first-order necessary condition equation (28) is derived in Step 3 of Lemma 1. Substituting for $a_{j,i}(z)$ with equation (8), we arrive at the equation in the statement of the lemma.

A.3 Proof of Lemma 3:

Suppose the government does not trade in any asset market, i.e., $a_G(z) \equiv 0$. Then equation 9 reduces to

$$\Lambda_{j,i}(z) - q(z) = \frac{\mu}{m_f} q'(z) \Big(a_i(z) \Big). \tag{33}$$

In the competitive equilibrium where $\frac{\mu}{m_f} q'(z) = 0$, both buyers and sellers optimally equate their state prices with asset prices ($\Lambda_{j,i}(z) = q(z)$).

[12] This result is true for any full-rank asset span X. See, for instance, Carvajal (2018) and Neuhann and Sockin (2022). Neuhann and Sockin (2022) show that these results hold more generally with incomplete markets among equivalent asset spans.

Let us conjecture that asset prices are the same in the Cournot-Walras Equilibrium as in the competitive equilibrium. However, according to equation 33, a buyer in asset market z will leave her state price above this asset price ($\Lambda_{j,i}(z) > q(z)$)). As such, she buys less than she would in the competitive equilibrium. Similarly, if she is a seller, then she leaves her state price below the asset price ($\Lambda_{j,i}(z) < q(z)$)). As such, she sells less than she would in the competitive equilibrium.

As this is true market-by-market, we consequently conclude that there is less trade in the Cournot-Walras than in the competitive equilibrium.

A.4 Proof of Proposition 4:

Step 1: Wealth-neutral Demand Shock with Perfect Competition:

Let us conjecture that asset prices are unaffected by the demand shock.

With perfect competition (i..e, $\mu = 0$) and complete markets, large agents of type i maximizes their utility from consumption subject to the inter-temporal budget constraint

$$\sum_{z\in\mathcal{Z}} q(z)\, y_i(z) + w_i = c_{1,j,i} + \sum_{z\in\mathcal{Z}} q(z)\, c_{2,j,i}(z). \tag{34}$$

Since the demand shock is *wealth-neutral*, $\sum_{z\in\mathcal{Z}} q(z)\, y_i(z)$ is the same as before the demand shock if asset prices are unchanged. Because the present-value of their endowments is unchanged under a wealth-neutral demand shock, as are the aggregate resources available in each state $\sum_{i=1}^{N} y_i(z)$, their problem is unchanged. Consequently, they choose the same consumption allocations that they did before the demand shock.

If all large agents' consumption allocations are unchanged, then so is the competitive fringe's by market clearing. If the fringe's consumption is unchanged, then so are asset prices, which are equal to the fringe's marginal utility in each state. This confirms the conjecture and the neutrality of *wealth-neutral* demand shocks for asset prices and consumption allocations.

Step 2: Wealth-neutral Demand Shock with Imperfectly Competition:

With market concentration, large agents of type i instead maximize their utility from consumption subject to the inter-temporal budget constraint

$$\sum_{z \in \mathcal{Z}} \Lambda_{j,i}(z) \, y_i(z) + w_i = c_{1,j,i} + \sum_{z \in \mathcal{Z}} \Lambda_{j,i}(z) \, c_{2,j,i}(z) \,. \tag{35}$$

Importantly, although the demand shocks is *wealth-neutral* under market prices, it is not under type i's state prices. Consequently, the demand shock is not neutral for their consumption allocation decision, and type i agents choose a different consumption bundle.

If all agents choose different consumption bundles after the demand shock, then generically so will the competitive fringe. If the fringe's consumption changes, then so do asset prices. A *wealth-neutral* demand shock is therefore not neutral with market concentration.

Step 3: Elasticity of Consumption and Asset Prices:

It is immediate because consumption allocations are altered by the demand shock, but are invariant in the perfect competition benchmark, that the elasticity of consumption with respect to demand shocks is lower with imperfect competition. This is because market concentration induces both sellers and buyers of Arrow securities to trade less, which deters reallocation of the demand shocks through financial markets.

Similarly, because asset prices change with market concentration but are invariant in the perfect competition limit, the elasticity of asset prices with respect to demand shocks is higher with imperfect competition.

Step 4: Dispersion in State Prices:

Suppose there is a wealth-neutral demand shock that increases the endowments of sellers in a given state and reduces the endowment of buyers. Then, because this raises the state prices of buyers and lowers it of sellers, and is only imperfectly reallocated, the dispersion of state prices, as measured according to $\omega(z)$, rises.

A.5 Proof of Proposition 1:

Step 1: The Case of the Competitive Equilibrium:

When all agents behave competitively, and $\mu = 0$, equation (9) reduces to:

$$q(z) = \frac{\pi(z) u'(c_{2,i}(z))}{u'(c_{1,i})} = \pi(z) u'_f(c_f(z)) = \Lambda^{CE}(z), \tag{36}$$

and all agents align their state prices state-by-state. There is therefore perfect risk sharing in the competitive equilibrium. This implies for the N types of agents with homothetic preferences:

$$\frac{c_{2,i}(z)}{c_{1,i}} = \frac{c_{2,j}(z)}{c_{1,j}} = \eta(z), \tag{37}$$

and for the competitive fringe:

$$c_f(z) = \eta_f(z) = u_f^{-1}(u'(\eta(z))). \tag{38}$$

Substituting for date 2 consumption into the budget constraint at date 1, the intertemporal budget constraint for agents of type i is:

$$c_{1,i} + \sum_{z \in \mathcal{Z}} q(z) c_{2i}(z) = w - \tau_1 + \sum_{z \in \mathcal{Z}} q(z)(y_i(z) - \tau_2(z)). \tag{39}$$

Recognizing that $\tau_1 = -\sum_{z \in \mathcal{Z}} q(z) \tau_2(z)$, it follows that equation (39) reduces to:

$$c_{1,i} + \sum_{z \in Z} q(z) c_{2i}(z) = w + \sum_{z \in \mathcal{Z}} q(z) y_i(z). \tag{40}$$

Finally, substituting with equation (51), we arrive at:

$$c_{1,i} = \frac{w + \sum_{z \in \mathcal{Z}} q(z) y_i(z)}{1 + \sum_{z \in \mathcal{Z}} q(z) \eta(z)}. \tag{41}$$

Suppose the consumption of the fringe is invariant to the government's policies, then so is $q(z)$ because it is equal to the state prices of the fringe. If prices are unchanged, then from equation (41) $c_{1,i}$ is also unchanged, and so are $c_{2,i}(z) = \eta(z) c_{1,i}$. By market-clearing then, so is the consumption of the fringe, confirming the conjecture.

The government's trading activity is therefore irrelevant for all agents and there is Ricardian Equivalence. This is because agents can frictionlessly transfer wealth across both states and dates.

Step 2: The Case of the Cournot-Walras Equilibrium:

First, we recognize that equation (39) remains valid when large agents are strategic. As such, there are no direct effects from government trading in the Cournot-Walras equilibrium. Notice from equation (28) that $q(z) = \Lambda_i(z) - \frac{\mu}{m_f} q'(z) a_i(z)$. Substituting this into equation (39), we arrive at:

$$c_{1,i} + \sum_{z \in \mathcal{Z}} \Lambda_i(z) c_{2i}(z) = w + \sum_{z \in \mathcal{Z}} \Lambda_i(z) y_i(z) + \frac{\mu}{m_f} \sum_{z \in \mathcal{Z}} q'(z) a_i(z)^2 . \tag{42}$$

The last term on the right-hand side of equation (42) is zero in the competitive equilibrium. This reveals that in the Cournot-Walras equilibrium, asset positions $a_i(z)$ are not irrelevant for equilibrium allocations. As such, trading away from the effective endowment of $-\frac{1}{N+m_f} a_g$ in assets from the government transfers is not frictionless.

Notice now the Cournot-Walras equilibrium features too little trading relative to the competitive benchmark by all agents, based on the implied wedges between state prices and asset prices in equation (28). It then follows that the competitive fringe absorbs at least part of the government's trades. As a result, if the government sells risk-free debt $a_g < 0$, then the fringe has more consumption at date 2, all else equal. This reduces its marginal utility in all states and all Arrow asset prices. In contrast, if the government buys risk-free debt $a_g > 0$, then the fringe has less consumption. This raises marginal utility in all states and all Arrow asset prices.

A.6 Proof of Proposition 2:

We first recognize that in the type-symmetric case, all agents consume the same initial consumption $c_{1,j,i}$ at date 1. Further, in the limit $m_f \to 0$ and $\frac{\mu}{m_f} = \kappa$,

$$\sum_{i=1}^{N} \hat{a}_i(z) = 0, \tag{43}$$

and it must be the case that $c_{1,j,i} = w$, i.e., agents consume their initial wealth. Consequently, the state price of agent j of type i is $\Lambda_{j,i}(z) = \frac{u'\left(y_i(z) + \hat{a}_{j,i}(z)\right)}{u'(w)}$. Consequently, the state prices of strategic agents move inversely with their asset positions $\hat{a}_{j,i}(z)$ state-by-state. In addition, because agents are type-symmetric, we need only focus on characterizing one asset market because they each behave identically for all z.

We next consider the seller side of asset market z. It is immediate from equations (9) and Proposition 1 that when $a_g < 0$ that sellers in asset market z sell more (i.e., $\hat{a}_{j,i}(z)$ becomes more negative. This is because a negative a_g effectively endows each agent with a long position that they must undo in financial markets and because a negative a_g lowers asset prices and consequently price impact. Both forces reduce the market concentration wedge between state prices $\Lambda_{j,i}(z)$ and asset prices $q(z)$.

Because an increase in normalized asset sales by seller j of type i raises her state price in state z, all sellers' state prices in asset market z rise. Since sellers have lower state prices than buyers (by definition of how agents sort into both sides of financial markets), government asset sales $a_g < 0$ decreases the dispersion in state prices referencing state z $\omega(z)$. By the converse argument, government asset purchases $a_g > 0$ instead raise this dispersion.

Finally, we consider the buyer side of asset market z. Although lower prices and price impact increase a buyer's normalized asset purchases from equation (9), the endowment of buyers with a long position $-\frac{a_g}{N+m_f}$ instead reduces her position. However, we recognize by market-clearing in the limit $m_f \to 0$ and $\frac{\mu}{m_f} = \kappa$ that

$$\sum_{i=1}^{N} \hat{a}_i(z) = 0. \tag{44}$$

Because each seller increases her sales in market z, it must be the case that buyers, on net, buy more assets based on their normalized demand. This lowers the state prices $\Lambda_{j,i'}(z)$ for those buyers who buy more, which further reduces the dispersion in state prices referencing state z, $\omega(z)$.

If all buyers increase their normalized demands, then $\omega(z)$ falls for all z. Otherwise, notice in the cross-section, strategic agents who buy the least (i.e., smallest $\hat{a}_{j,i}(z)$) have the lowest state prices among buyers, while those that buy the most (i.e., largest $\hat{a}_{j,i}(z)$) have the highest state prices. Consequently, a decline in price impact because of government sales must increase the normalized asset demand of the high state price buyers, and lower their state prices, and decrease the normalized asset demand of low state price buyers, and raise their state prices. Because lower state price buyers have more elastic demand than high state price buyers, it follows the overall effect is to decrease the dispersion in state prices ω_z.

An analogous argument establishes that asset purchases $a_g > 0$ raise the state-price dispersion ω_z. Consequently, ω_z is increasing in a_g.

A.7 Proof of Proposition 3:

We begin with the Euler Equations from equation (9) expressed in terms of risk sharing wedges $\Lambda_{j,i}(z) - q(z)$ and the normalized asset demands $\hat{a}_{j,i}$, which we write as

$$\Lambda_{j,i}(z) - q(z) = \frac{\mu}{m_f} q'(z) \hat{a}_{j,i}(z) - \frac{\mu}{m_f} q'(z) \frac{a_g}{N + m_f}, \tag{45}$$

and depends on $\frac{a_g}{N+m_f}$ only through the last term on the right-hand side. If $a_g > 0$, then the last term is negative, while if $a_g < 0$, then the last term is positive.

Notice government asset purchases cannot generically achieve the competitive equilibrium. This is because a_g (which is one degree of freedom) cannot be chosen such that the wedges $\frac{\mu}{m_f} q'(z) \left(\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f} \right)$ ($N \times |\mathcal{Z}|$ equations) are all zero.[13]

In what follows, we measure the aggregate efficiency of risk sharing in an asset market using $Var_i(\Lambda_i(z))$ defined in equation (11). The change in efficiency in risk sharing in state z for a change in government policy a_g is

$$\Delta \log Var_i(\Lambda_i(z)) = 2\Delta \log q'(z) + \Delta \log Var_i(\hat{a}_i(z)). \tag{46}$$

From Proposition 1, $\frac{\partial q'(z)}{\partial a_g} > 0$ and the first-term in equation (47) is positive if $a_g > 0$ and negative if $a_g < 0$ for all z. Because sellers' supply curves are more elastic than buyers' demand curves, it follows $\frac{\partial}{\partial a_g} Var_i(\hat{a}_i(z)) < 0$ because sellers reduce their selling positions more than buyers increase their buying positions.

We can further approximate equation (46) to first-order for a change in government

[13]This is also true even if the government can trade each Arrow asset separately because there are still more Euler Equations than asset markets. Because prices (and consequently price impact) cannot be zero by no arbitrage, these wedge will not vanish if government asset sales are arbitrarily large.

policy Δa_g as

$$\Delta \log Var_i \left(\Lambda_i \left(z\right)\right) \approx 2\frac{\gamma_f \left(z\right)}{m_f} \sum_{i=1}^{N} \Delta \hat{a}_i \left(z\right) + 2\sum_{i=1}^{N} w_i \frac{\Delta \hat{a}_i \left(z\right)}{\hat{a}_i \left(z\right)}, \tag{47}$$

where $\gamma_f \left(z\right)$ is the competitive fringe's coefficient of absolute risk aversion in state z and $w_i = \frac{1}{N} \frac{\hat{a}_i(z)^2 - \hat{a}_i(z) \frac{1}{N} \sum_{j=1}^{N} \hat{a}_j(z)}{Var_i(\hat{a}_i(z))}$ are bounded weights that sum to 1 with the convention that $w_i \hat{a}_i \left(z\right) = -\frac{1}{N} \frac{\frac{1}{N} \sum_{j=1}^{N} \hat{a}_j(z)}{Var_i(\hat{a}_i(z))}$ when $\hat{a}_i \left(z\right) = 0$.

To first-order, the change in the efficiency of risk sharing is driven by how the change in each agent's net asset position $\hat{a}_i \left(z\right)$ impacts not only market liquidity but also their position relative to the mean exposure $\frac{1}{N} \sum_{j=1}^{N} \hat{a}_j \left(z\right)$. The first term is increasing in $\frac{\gamma(z)}{m_f}$, which is the inverse of the effective risk-bearing capacity of the fringe. Let $\underline{\gamma} = \min_{z \in \mathcal{Z}} \gamma \left(z\right)$, i.e., the minimum coefficient of risk aversion across all asset markets.

Step 1: Asymptotic Absorption Capacity:

Consider first large government asset purchases $a_g >> 0$. Such a large purchase induces sellers in each market to ration severely their supply in each market (even becoming buyers), and buyers to increase their demand. This demand is absorbed by the fringe until $\sum_{i=1}^{N} \hat{a}_i \left(z\right) = m_f y_f \left(z\right)$, in which case the asset price in market z $q(z)$ becomes infinite, as does price impact. This immiseration pushes $Var_i \left(\Lambda_i \left(z\right)\right)$ to its autarky value. Consequently, sufficiently large asset purchases severely worsen the efficiency of risk sharing.

Consider next large government asset sales $a_g << 0$. In this case, the market is saturated with supply of each asset and prices and price impact are very low from Proposition 1. Because the wedge is bounded from below by 0, and from above because state prices cannot become infinite since strategic agent utility satisfies the Inada condition, the efficiency measure eventually asymptotes. Notice now from equation (45) that for the measure to asymptote, it must be the case that $\lim_{a_g \to -\infty} q'(z) a_g = 0$. For this to be the case, $q'(z)$ must fall faster than $|a_g|$ rises (in fact, $q'(z)$ is convex in a_g). As such, $Var_i \left(\Lambda_i \left(z\right)\right)$ asymptotically decreases to its limit. Because from above, government asset purchases cannot achieve the competitive equilibrium, it follows this limit is bounded away from 0.

Consequently, when a_g is arbitrarily negative, $Var_i(\Lambda_i(z))$ converges to its lower limit above 0, while when it is positive and too large, $Var_i(\Lambda_i(z))$ converges to its autarky value. Because $Var_i(\Lambda_i(z))$ is continuous in a_g, it follows $Var_i(\Lambda_i(z))$ has an even number of turning points in a_g. Given $Var_i(\Lambda_i(z))$ is driven by two monotonic forces that move in opposite directions, price impact and asset position variance, there are either zero or two turning points.

Step 2: Small Government Asset Purchases $a_g > 0$:

Suppose the government purchases a small amount of assets, i.e., $a_g > 0$. Because $a_g > 0$, the overall wedge for asset sellers $\frac{\mu}{m_f} q'(z) \left(\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f} \right)$ increases, and consequently they trade less. As a result, asset buyers buy more from the fringe (which is why prices increase). In contrast, the wedge may increase or decrease (or even become negative) for buyers depending on whether the increase in price impact is offset by the decrease in $\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f}$. Consequently, the risk sharing wedge unambiguously worsens for sellers, but may improve or worsen for buyers.

Consider now the first-order change in risk sharing efficiency (equation (47)) of a small increase in a_g from 0 to $\Delta a_g > 0$. From our above discussion, the first piece is positive for all z while the second piece is negative for all z. If $\frac{\gamma(z)}{m_f}$ is sufficiently large, then the price impact dominates the position variance effect, and the efficiency measure rises.

Step 3: Small Government Asset Sales $a_g < 0$:

Suppose the government sells a small amount of assets, i.e., $a_g < 0$. Because $a_g < 0$, the overall wedge for asset sellers $\frac{\mu}{m_f} q'(z) \left(\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f} \right)$ decreases, and consequently they sell more. In contrast, the wedge may increase or decrease (or even become positive) for buyers depending on whether the decrease in price impact is offset by the increase in $\hat{a}_{j,i}(z) - \frac{a_g}{N+m_f}$. Consequently, risk sharing unambiguously improves for sellers, but may improve or worsen for buyers.

Consider now the first-order change in risk sharing efficiency (equation (47)) of a small increase in a_g from 0 to $\Delta a_g > 0$. From our above discussion, the first piece is positive for all z while the second piece is negative for all z. If $\frac{\gamma(z)}{m_f}$ is sufficiently large, then the price impact dominates the position variance effect, and the efficiency measure falls.

A.8 Proof of Proposition 4:

We first consider how unfunded government purchases $\bar{a}_g > 0$ impact equilibrium allocations in the case of perfect competition. We then consider the complementary case with concentrated financial markets.

Step 1: Perfect Competition:

With perfect competition, it is immediate that the First Welfare Theorem holds and optimal risk-sharing arrangements solve the appropriate social planner's problem. In this case, with symmetric, homothetic preferences, perfect risk sharing calls for $\frac{c_{2i}(z)}{c_{1i}} = \eta(z) = u'^{-1}(q(z))$ for all i. This is Wilson's optimal risk sharing rule. In this case, $\sum_{z\in\mathcal{Z}} q(z) = \sum_{z\in\mathcal{Z}} u'(\eta(z))$

By the inter-temporal budget constraint for agent i at date 1

$$c_{1i} + \sum_{z\in\mathcal{Z}} q(z)\, c_{2i}(z) = \left(1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)\right) c_{1i} = w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z), \tag{48}$$

from which follows

$$c_{1i} = \frac{w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z)}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)}. \tag{49}$$

Further, by market clearing in the consumption market at date 2 in the strategic limit in which $m_f \to 0$

$$\sum_{i=1}^{N} c_{2i}(z) + \bar{a}_g = \eta(z) \sum_{i=1}^{N} c_{1i} + \bar{a}_g = \sum_{i=1}^{N} y_i(z), \tag{50}$$

from which follows from equation (49) and $a_g = \frac{y_{1g} - y_{2g}}{1 + \sum_{z\in\mathcal{Z}} q(z)}$ that $\eta(z)$ solve

$$\eta(z) \sum_{i=1}^{N} \frac{w_i + \sum_{z\in\mathcal{Z}} u'(\eta(z))\, y_i(z)}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))\,\eta(z)} + \frac{\Delta y_g}{1 + \sum_{z\in\mathcal{Z}} u'(\eta(z))} = \sum_{i=1}^{N} y_i(z). \tag{51}$$

Recovering $\eta(z)$ from equation (51) is consequently sufficient to solve for the competitive equilibrium.

Notice because $\Delta y_g = y_{1g} - y_{2g} > 0$ (i.e., $\bar{a}_g > 0$) and $u'(\eta(z))$ is decreasing in

$\eta(z)$ from applying the Implicit Function Theorem to equation (51) that $\frac{\partial \eta(z)}{\partial \Delta y_g} < 0$. In addition, by the Chain Rule, $\frac{\partial q(z)}{\partial \Delta y_g} = \frac{\partial q(z)}{\partial \eta(z)} \frac{\partial \eta(z)}{\partial \Delta y_g} > 0$ for all $z \in \mathcal{Z}$ because $\frac{\partial q(z)}{\partial \eta(z)} < 0$ with convex marginal utility.

Consequently, as $\Delta y_g > 0$ increases, all agents consume more at date 1 and less at date 2, and asset prices rise state-by-state. Because the government crowds out investment in risk-free bonds, agents are forced to consume more resources at date 1.

Step 2: Imperfect Competition:

Notice in the limit $m_f \to 0$ that asset prices are given by the average of strategic agents' state prices, $q(z) = \frac{1}{N} \sum_{i=1}^{N} \Lambda_i(z)$. Suppose that Δy_g increases to $\Delta y_g + \varepsilon_g$. We conjecture that $\bar{a}_g$ increases and, based on Step 1, all asset prices fall and strategic agents consume more at date 1 and less at date 2, state-by-state. In this case, all strategic agents' state prices (weakly) rise state-by-state, $\Lambda_i(z)$, which raises asset prices and lowers the risk-free rate. Because the risk-free rate falls, the government has to (weakly) buy more assets to achieve its pre-existing date 2 position based on Δy_g, as well as more to cover its incremental position based on ε_g. Therefore, $\bar{a}_g$ increases, confirming the conjecture.

In addition to the direct effect of government purchases, the increase in asset prices raises price impact $q'(z)$ because the competitive fringe's marginal utility is convex. This further attenuates how much risk-free debt is traded among strategic agents. As a result, buyers reduce their purchases of risk-free debt more than with perfect competition, while sellers must sell to fulfill the government's incremental orders, exacerbating the rise in asset prices.

Consequently, as with perfect competition, asset prices rise and strategic agents consume more at date 1 and less at date 2.

A.9 Proof of Proposition 5:

Consider a perturbation to the welfare objective (20) from increasing the government's debt position, a_G. This has two effects. First, mechanically, it marginally shifts all agents' resources from date 1 to date 2. This is because taxes increase at date 1 and are more negative at date 2 to finance the purchase of the debt and lump-sum rebate of its proceeds.

Second, it marginally increases the price of every Arrow asset from the government's increased demand. To first-order, the change in each agent's asset position on her own utility is zero by the Envelope Theorem applied to their respective optimization problems from Proposition (1). There is, however, a wealth effect on each agent's utility from having to buy a more expensive asset portfolio, and a strategic indirect effect that a change in large agents' asset positions affects the price impact of other large agents.

Perturbing in utilitarian welfare by altering Δa_g reveals:

$$\Delta W(h) = \sum_{i=1}^{N} \theta_i u'(c_{1,i}) \left(\Delta c_{1,i} + \sum_{z \in \mathcal{Z}} \Lambda_i(z) \Delta c_{2,i}(z) \right) + \theta_f m_f \left(\sum_{z \in \mathcal{Z}} u_f'(c_{2,f}(z)) \Delta c_{2,f}(z) - \Delta c_{1,f} \right), \tag{52}$$

which we can expand with equations (4) (5), and (8)

$$\begin{aligned} \frac{\Delta W}{\Delta a_g} &= \sum_{i=1}^{N} \theta_i u'(c_{1,i}) \sum_{z \in \mathcal{Z}} \left((\Lambda_i(z) - q(z)) \frac{\Delta \hat{a}_i(z)}{\Delta a_g} - \frac{\Delta q(z)}{\Delta a_g} \hat{a}_i(z) \right) \\ &+ \theta_f m_f \left(\sum_{z \in \mathcal{Z}} \left(u_f'(c_{2,f}(z)) - q(z) \right) \frac{\Delta \hat{a}_f(z)}{\Delta a_g} - \frac{\Delta q(z)}{\Delta a_g} \hat{a}_f(z) \right), \end{aligned} \tag{53}$$

where $\frac{\Delta q(z)}{\Delta a_g}$ indicates the total change in $q(z)$ with respect to a_g. Notice that the first m_f term in equation (53) is zero by the definition of the Arrow price $q(z)$ and $m_f \hat{a}_f = -\sum_{i=1}^{N} \hat{a}_i(z)$. Equations (53) and (28) consequently reduces to:

$$\frac{\Delta W}{\Delta a_g} = \sum_{z \in \mathcal{Z}} \frac{\mu}{m_f} q'(z) \sum_{i=1}^{N} \theta_i u'(c_{1,i}) a_i(z) \frac{\Delta \hat{a}_i(z)}{\Delta a_g} - \frac{\Delta q(z)}{\Delta a_g} \sum_{i=1}^{N} \left(\theta_i u'(c_{1,i}) - \theta_f \right) \hat{a}_i(z). \tag{54}$$

Taking the limit of equation (54) as $\Delta a_g \to da_g$, defining $w_p(z) = \sum_{i=1}^{N} \theta_i u'(c_{1,i}) a_i(z) \frac{d\hat{a}_i(z)}{da_g}$ and $w_g(z) = \sum_{i=1}^{N} \left(\theta_i u'(c_{1,i}) - \theta_f \right) \hat{a}_i(z)$, and recognizing that a necessary condition for optimality is $\frac{\partial W}{\partial a_g} = 0$, at the optimal a_g it must be the case from (54):

$$\sum_{z \in \mathcal{Z}} w_p(z) \frac{\mu}{m_f} q'(z) - w_g(z) \frac{dq(z)}{da_g} = 0 \; (< if a_g = -\infty, > if a_g = \infty) \tag{55}$$

If all large agents behaved competitively, then $\frac{\mu}{m_f} q'(z) = 0$, and because of Ricardian Equivalence, $\frac{dq(z)}{da_g} = 0$ from from Proposition 1. Therefore, the first-order condition reduces to 0 in the competitive equilibrium, and is trivially satisfied.