Demand-System Asset Pricing: Theoretical Foundations

(plus a few new results)

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2. Use investor-specific "tastes" for financial assets to account for portfolio heterogeneity.

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We study the foundations and interpretation of this methodology.

A brief history of thought

1. "Neoclassical asset pricing:" focus on relative prices, quantities mostly irrelevant.

2. Demand effects a la index inclusion: a notion of an *aggregate* demand curve.
3. High-frequency identification of aggregate effects of QE.
4. Intermediary asset pricing.

5. Asset demand systems: **structurally estimate** individual- and asset-level demand on portfolio data.

Existing methods hew closely to IO, but financial assets present unique challenges

1. Central role of **portfolios** and **relative prices** rather than good-specific demand functions.

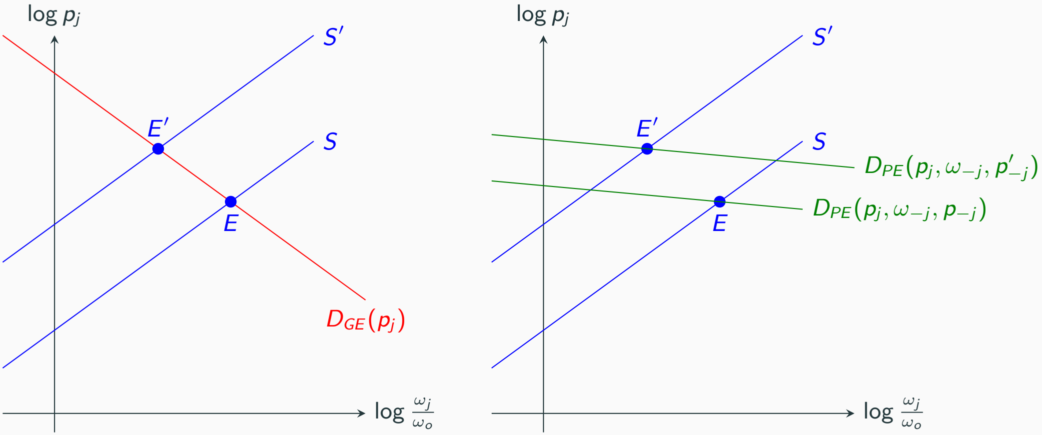
2. Cross-asset price spillovers through **general equilibrium price determination** and **no arbitrage**.

3. **Resale** considerations: current demand depends on future prices.

Today

1. Cross-asset portfolio linkages and price spillovers can heavily bias measured elasticities.
    - In current frameworks, measured elasticities may be near zero even if true elasticities are near infinite.
    - Control variables do not address this issue – in fact, may be quite uninformative.
2. A general tension between demand estimation and no arbitrage.
    - *Ceteris paribus* thought experiment hard to estimate from arbitrage-free markets data.
3. Elasticities are to be interpreted as structural objects only under strong additional assumptions.
    - Identification challenges: preferences versus latent constraints; Keynesian beauty contests.

Summary graph



(a) Logit demand

(b) Demand with cross-asset complementarities

Framework

Basic considerations

We want a simple theoretical laboratory to assess demand estimation.

To align with current approach, we allow heterogeneous *tastes* for financial assets.

Paper analyzes this in more detail.

1. Taste disagreement is critical for identification – need mutually orthogonal demand shocks.
2. This disagreement can lead to violations of no arbitrage – precisely because valuations differ.

   Practical concern because most applications eventually estimate reduced-form specifications.

For today, will simply assume that I have access to a clean asset-level supply shock.

Framework

A one-shot portfolio choice problem:

- Investor i can choose consumption at date 0 or at date 1. Assume log utility.
- State $z \in \mathcal{Z} \equiv \{1, \ldots, Z\}$ with probability $\pi_z \in (0, 1)$.
- Assets $\mathcal{J} \equiv \{1, \ldots, J\}$ with price p_j and state-contingent cash flows $y_j(z)$.
- A portfolio of assets $(a_j^i)_{j \in \mathcal{J}}$.
- Investors receive asset endowments e_j^i and non-marketable endowment w_0^i and $w_1^i(z)$.

Incorporating tastes

Given asset-specific taste parameter θ_j^i, investor i evaluates asset j's payoff $y_j(z)$ as $\theta_j^i y_j(z)$.

Then define utility over **taste-adjusted consumption**

$$\tilde{c}_1^i(z) \equiv \sum_j \theta_j^i y_j(z) a_j^i + w_1^i(z).$$

This allows us to use the standard machinery of expected utility. Moreover:

- If we let $\theta_j^i = 1$ for all j we recover the standard model.
- Close connection to dogmatic belief over the scale of the payoff.

Taste-augmented portfolio choice problem

Our approach leads to a simple generalization of the standard problem:

$$\max_{(a_1^i, a_2^i, \dots, a_J^i)} \quad (1-\delta)u(c_0^i) + \delta \sum_z \pi_z u\Big(\tilde{c}^i(z)\Big) + \text{continuation value}$$

$$\text{s.t.} \quad \tilde{c}_1^i(z) \equiv \sum_j \theta_j^i y_j(z) a_j^i + w_1^i(z),$$

$$c_0^i + \sum p_j(a_j^i - e_j^i) = w_0^i,$$

ad-hoc portfolio restrictions or mandates.

We model tastes over **payoffs**, not returns. **This allows for endogenous return spillovers.**

Identifying asset-level demand elasticities

What is an asset-level demand function?

Portfolio choice models generate predictions for quantities to be bought of any given asset, say

$$a^i_j(\vec{p}) : \qquad \text{a function of the vector of all asset prices}$$

Asset-level demand functions can be described using the notation of a **demand elasticity**.

This reflects **thought experiment** in which we vary a single asset price – that is, a **partial derivative**:

$$\mathcal{E}^i_{js} \equiv -\frac{\partial a^i_j(\vec{p})}{\partial p_s} \frac{p_s}{a^i_j(p}.$$

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Identification problem: given shock χ_s to asset s, observational data shows only the **total derivative**:

$$\hat{\mathcal{E}}^i_{ss} \equiv -\frac{\frac{da^i_j}{d\chi_s}}{\frac{dp_s}{d\chi_s}} \cdot \frac{p_s}{a^i_j}.$$

Deriving demand functions

The first-order necessary condition for the optimal choice of with respect to a_j^i is:

$$\underbrace{\theta_j^i \sum_{z\in\mathcal{Z}} y_j(z) \frac{u^{i'}(\tilde{c}^i(z))}{u^{i'}(c_0^i)} \quad + \quad \texttt{Lagrange multipliers} \quad - \quad p_j}_{\equiv \text{ Net marginal value } F_j^i(a^i,\, p)} = 0.$$

Deriving demand functions

Asset-level demand functions are jointly determined by a **system of equations** (+ constraints):

$$\begin{bmatrix} F_1^i(a^i, p) \\ F_2^i(a^i, p) \\ \vdots \\ F_J^i(a^i, p) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

This system typically exhibits **demand complementarities**: marginal value of asset j depends on a^i_{-j}.

1. Diversification: marginal value depends on covariance with portfolio.
2. Constraints: investment mandates which allow substitution between some assets.

Response to an exogenous supply shock χ_s to asset s.

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Total derivative of i's portfolio in response to the shock is:

$$\begin{bmatrix} \frac{da_1^i}{d\chi_s} \\ \vdots \\ \frac{da_s^i}{d\chi_s} \\ \vdots \\ \frac{da_J^i}{d\chi_s} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial a_1^i}{\partial p_1} & \cdots & \frac{\partial a_1^i}{\partial p_s} & \cdots & \frac{\partial a_1^i}{\partial p_J} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial a_s^i}{\partial p_1} & \cdots & \frac{\partial a_s^i}{\partial p_s} & \cdots & \frac{\partial a_s^i}{\partial p_J} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial a_J^i}{\partial p_1} & \cdots & \frac{\partial a_J^i}{\partial p_s} & \cdots & \frac{\partial a_J^i}{\partial p_J} \end{bmatrix}}_{\text{Quantity responses}} \underbrace{\begin{bmatrix} \frac{dp_1}{d\chi_s} \\ \vdots \\ \frac{dp_s}{d\chi_s} \\ \vdots \\ \frac{dp_J}{d\chi_s} \end{bmatrix}}_{\text{Price spillovers}} + \underbrace{\begin{bmatrix} \frac{\partial a_1^i}{\partial \chi_s} \\ \vdots \\ \frac{\partial a_s^i}{\partial \chi_s} \\ \vdots \\ \frac{\partial a_J^i}{\partial \chi_s} \end{bmatrix}}_{\text{Income effects}} .$$

Structural elasticities obscured by cross-asset interactions and equilibrium price spillovers $\mathcal{S}_{js} \equiv \frac{dp_j}{d\chi_s}$.

Observed versus structural elasticity

Proposition: The observed own-price elasticity $\hat{\mathcal{E}}^i_{ss}$ can be decomposed as follows:

$$\hat{\mathcal{E}}^i_{ss} = \mathcal{E}^i_{ss} - \underbrace{\sum_{j \neq s} \frac{\mathcal{S}_{js} \frac{1}{p_j}}{\frac{dp_s}{d\chi_s} \frac{1}{p_s}} \mathcal{E}^i_{sj}}_{\text{Complementarities}} - \underbrace{\frac{\frac{\partial a^i_s}{\partial \chi_s} \frac{1}{a^i_s}}{\frac{dp_s}{d\chi_s} \frac{1}{p_s}}}_{\text{Income effects}} . \quad (1)$$

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Intuition in a simple example. Investor chooses between bond and two similar stocks.

1. Holding other price fixed, a price change triggers rapid reallocation to other stock ($\mathcal{E}^i_{sj}$ is large).
2. If many investors attempt to do this, other price must adjust.
3. Once prices adjust, there is no need to adjust your portfolio.

How to isolate the structural elasticity from the observed one?

Generic answer: one must place restrictions on the substitution matrix.

Canonical IO: discrete choice over goods with homogeneous substitution to an "outside good."

KY use a similar system: "logit" demand for financial assets conditional on characteristics.

1. Impose assumptions on *returns* to sharply reduce scope for cross-asset spillovers.
2. Yields homogeneous substitution in units of portfolio weights relative to an outside asset.

The KY logit demand framework

Assume equilibrium **returns** follow a factor structure with **diagonal** conditional covariance matrix.

This means there are no complementarities left to worry about conditional on the factors.

Demand in units of relative portfolio weights assumed to depend only on own prices and characteristics:

$$\frac{\omega_j(p)}{\omega_{out}(p)} = \frac{\omega_j}{\omega_{out}}(p_j) = \exp\left\{\beta_0 \log p_j + \sum_{k=1}^{K-1} \beta_k x_k(j) + \beta_K\right\}\theta(j),$$

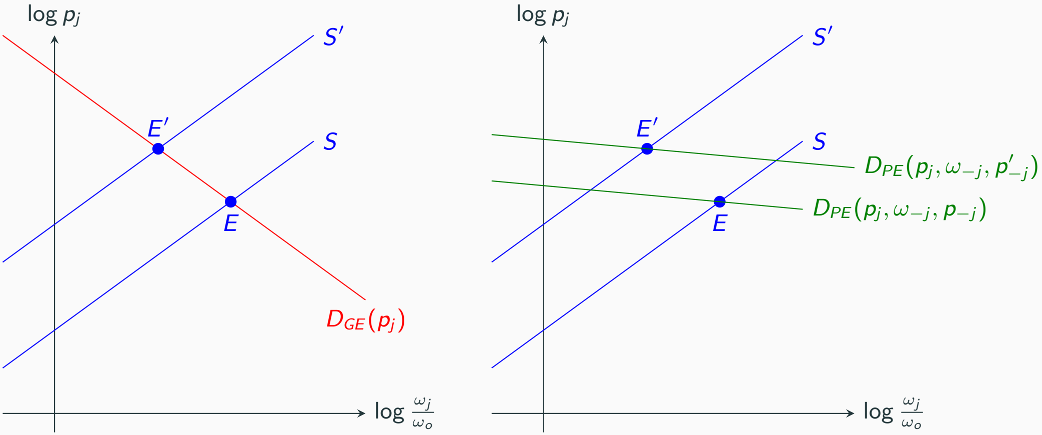
Identification: if $x_k(j)$'s are invariant to supply shocks, observed and structural elasticity are identical.

$\Rightarrow$ can identify β_0 from *observed* portfolio changes given price shocks.

$\Rightarrow$ because demand is separable across assets, need only one price instrument per asset.

Problem: Returns and substitution are **endogenous**. Does logit demand generate a factor structure?

Back to our graph



(a) Logit demand

(b) Demand with cross-asset complementarities

Do equilibrium returns and substitution patterns satisfy the logit structure?

Our approach

We derive equilibrium portfolio choices alongside returns in a canonical framework.

- In particular, enrich Lucas '78 with payoff-augmenting tastes and mandates.
- As in KY, use log utility for simplicity (but this isn't necessary).

Use this model to compute true and measured elasticities based on the logit structure.

A minimal asset menu

- Two aggregate states, $z = 1, 2$ with prob. $\pi_z = \frac{1}{2}$. For each z, one tree that pays 1 in z only.
- Split Tree 1 into **green** and **red** halves with diversifiable risk. Green half pays better in green state.

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- Parameter ϵ measures complementarity between assets. Tastes can be defined over colors: (θ^i_g, θ^i_r).
- Contrast with logit: heterogeneous substitution across assets if and only if $\epsilon < 1$.

Endowments and demand system implementation

- Tree 2 is the *outside asset* with normalized price $p_2 \equiv 1$. Relative prices p_r and p_g.
- Endowments $E_2 = 1$, $E_r = \frac{1}{2}$ and $E_g = \frac{1}{2} + \psi$. Use ψ as an **exogenous supply shock.**

- As in KY, define demand in units of portfolio shares relative to the outside good, $\frac{\omega_j^i(p)}{\omega_2^i(p)}$, so that

$$\mathcal{E}_{jj}^i \equiv -\frac{\partial(\omega_j^i(p)/\omega_2^i(p))}{\partial p_j}\frac{p_j}{\omega_j^i(p)/\omega_2^i(p)} \qquad \text{and} \qquad \hat{\mathcal{E}}_{jj}^i \equiv -\frac{d(\omega_j^i(p)/\omega_2^i(p))}{d p_j}\frac{p_j}{\omega_j^i(p)/\omega_2^i(p)}.$$

- NB: For the basic point, sufficient to assume no constraints or tastes over assets, $\theta_j^i = 1$.

Supply variation ψ is a clean instrument for p_g: fully exogenous to all investors in the model.

Under the hypothesis of logit demand, **structural elasticity = observed elasticity**, $\mathcal{E}^i_{jj} = \hat{\mathcal{E}}^i_{jj}$.

Proposition: Let $\mathcal{B}^i_{jj} \equiv \mathcal{E}^i_{jj} - \hat{\mathcal{E}}^i_{jj}$ denote the "logit bias." This bias is given by

$$\mathcal{B}^i_{jj} = -\underbrace{\frac{\partial\left(\omega^i_j(p)/\omega^i_2(p)\right)}{\partial p_{-j}}}_{\text{Complementarity}} \underbrace{\frac{p_{-j}}{\left(\omega^i_j(p)/\omega^i_2(p)\right)} \frac{p_j}{p_{-j}}}_{\text{Scaling terms}} \underbrace{\frac{dp_{-j}}{dp_j}}_{\text{Price Spillover}} .$$

Equilibrium demand functions

Contra logit, equilibrium demand functions **depend on both prices** for all $\epsilon < 1$.

$$\frac{\omega_g^i(p_g, p_r)}{\omega_2^i(p_g, p_r)} = \theta_r^i \frac{\pi_1}{\pi_2} p_g \cdot \frac{(\theta_r^i p_g + \theta_g^i p_r)\epsilon^2 - (\theta_r^i p_g - \theta_g^i p_r) + 2\theta_g^i p_r \epsilon(1 - 2\rho)}{(\theta_r^i p_g + \theta_g^i p_r)^2 \epsilon^2 - (\theta_r^i p_g - \theta_g p_r)^2}; \tag{2}$$

$$\frac{\omega_r^i(p_g, p_r)}{\omega_2^i(p_g, p_r)} = \theta_g^i \frac{\pi_1}{\pi_2} p_r \cdot \frac{(\theta_r^i p_g + \theta_g^i p_r)\epsilon^2 + (\theta_r^i p_g - \theta_g^i p_r) - 2\theta_r^i p_g \epsilon(1 - 2\rho)}{(\theta_r^i p_g + \theta_g^i p_r)^2 \epsilon^2 - (\theta_r^i p_g - \theta_g^i p_r)^2}. \tag{3}$$

Bias between observed and structural elasticity

Proposition. For a small shock to green supply ψ, the logit bias satisfies

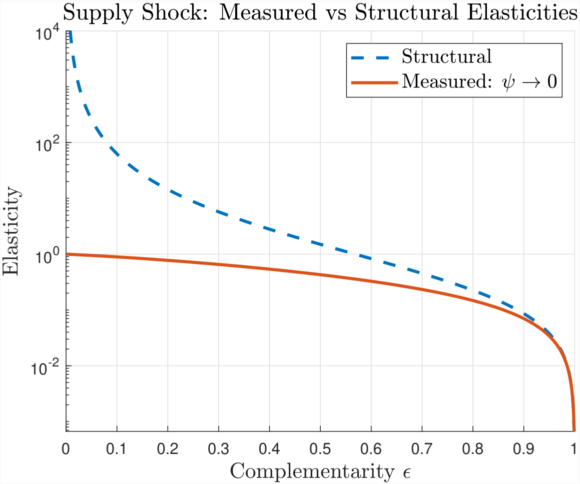
$$\mathcal{B}_{gg} = \frac{(1-\epsilon^2)^2(1+(1-2\rho)\epsilon)}{8\epsilon^2\rho(1-\rho)((1+\epsilon)^2-4\epsilon\rho)}.$$

The bias is positive, is strictly decreasing in ϵ, goes to infinity as $\epsilon \to 0$, and is zero iff $\epsilon = 1$.

In particular, in the limit as red and green assets become perfect substitutes, we have

$$\lim_{\epsilon\to 0} \mathcal{E}_{gg} = \infty \qquad \text{and} \qquad \lim_{\epsilon\to 0} \hat{\mathcal{E}}_{gg} = 1$$

Measured versus structural elasticities (log scale)



Measured is *always* low; true is low when $\epsilon \to 1$ $\quad (\rho = \frac{1}{4})$.

Interpretation

- When $\epsilon < 1$, substitution between red and green differs from substitution with Tree 2.
  Hence controlling for outside demand not sufficient to capture actual substitution patterns.

- With spillovers, observed elasticity is partly driven by state 1 *consumption elasticity.*
  Consumption elasticities can be low even if the asset-level elasticity is extremely high.

Koijen Yogo (2025) argue we used the wrong benchmark

Their abstract states:

> *[...] FFN25 use an incorrect [within-asset] estimator for their central claim that "measured elasticities are near one even if true elasticities are near infinite." The cross-sectional instrumental variables estimator correctly identifies the demand elasticities in KY19 and FFN25.*

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Their abstract states:

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The cross-sectional estimator they propose is:

$$\hat{\beta}_0 = -\frac{\mathrm{Cov}(\log \omega_j(\psi), z_j)}{\mathrm{Cov}(\log p_j(\psi), z_j)}.$$

What's going on?

In the model at hand, the cross-sectional estimator can be computed exactly:

$$\hat{\beta}_{0,\text{exact}} = -\frac{\log(\omega_g(\psi)) - \log(\omega_r(\psi))}{\log(p_g(\psi)) - \log(p_r(\psi))}.$$

It seems difficult to separate own- and cross-price elasticities from a single equilibrium allocation.

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It seems difficult to separate own- and cross-price elasticities from a single equilibrium allocation.

KY25 do not actually use the exact estimator. Instead, approximate demand and prices around $\psi = 0$,

$$\hat{\beta}_0 \approx \hat{\beta}_{0,\mathrm{approx}} \equiv -\frac{Cov\left(\frac{d \log \omega_j}{d\psi}\psi, z_j\right)}{Cov\left(\frac{d \log p_j}{d\psi}\psi, z_j\right)} = -\frac{\frac{d(\log(\omega_g(\psi)) - \log(\omega_r(\psi)))}{d\psi}}{\frac{d(\log(p_g(\psi)) - \log(p_r(\psi)))}{d\psi}}.$$

Since $\hat{\beta}_0 = F(\psi)/G(\psi)$, this is valid only if $F(0) = G(0) = 0$. This requires **perfect symmetry**, $\rho = \frac{1}{2}$.

Proposition (Bias in the cross-sectional estimator)

If $\rho \neq \frac{1}{2}$, the KY approximation is invalid and the exact value of the KY25 estimator $\hat{\beta}_0$ satisfies

$$\lim_{\psi \to 0} \hat{\beta}_{0,\text{exact}}(\psi) = -1,$$

which is of the opposite sign as, and does not vary with, the structural elasticity $\mathcal{E}_{gg}$.

If $\rho = \frac{1}{2}$, then the KY approximation is accurate and equation (B18) of KY25 holds,

$$\lim_{\psi \to 0} \hat{\beta}_{0,\text{exact}}(\psi) = \frac{1}{\epsilon^2} - 1.$$

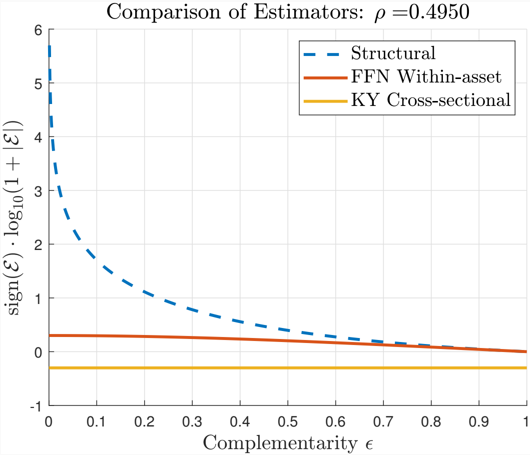

Figure 3: Estimators given a small violation of perfect symmetry ($\rho = 0.495$.) We use an approximate log scale to accommodate negative values.

The general tension

Asset-level demand estimation calls for all other prices to be held fixed.

But in arbitrage-free markets, **supply shocks** will lead to repricing of multiple assets.

Generic problem: the supply shock yields the wrong price variation. Only exception: Arrow securities.

No arbitrage algebra

If utility is defined over consumption, investors care about **state prices**, not asset prices.

State prices measure the cost of what the consumers care about.

Under no arbitrage, the state price induces by asset prices are proportional to the **inverse** payoff matrix

$$p = Yq \qquad \Leftrightarrow \qquad q = Y^{-1}p.$$

But supply shocks generically create state price variation that is proportional to Y. There exists $\mathbf{U}$ s.t.

$$\frac{\partial q}{\partial \psi_j} = -\mathbf{U}Y_j$$

Unless Y is diagonal, these are not proportional to each other, and need not even have the same signs!

Model: Hypothetical state price changes given shock to p_g

Let $\rho = \frac{1}{2}$. We can back out implied state prices from asset prices:

$$\begin{pmatrix} q_g \\ q_r \end{pmatrix} = \frac{1}{4\epsilon} \begin{pmatrix} (1+\epsilon)p_g - (1-\epsilon)p_r \\ -(1-\epsilon)p_g + (1+\epsilon)p_r \end{pmatrix}.$$

In the **thought experiment** where we vary p_g exogenously, induced state price changes are

$$\frac{\partial}{\partial p_g} \begin{pmatrix} q_g \\ q_r \end{pmatrix} = \frac{1}{4\epsilon} \begin{pmatrix} 1+\epsilon \\ -(1-\epsilon) \end{pmatrix}.$$

For any $\epsilon < 1$, **the green state becomes expensive and the red state becomes cheap.**

Model: Equilibrium state prices after a supply shock to g

Impose market clearing. Then **equilibrium state prices** satisfy

$$q_g^*(\psi) = \frac{1}{2} \cdot \frac{1}{1+\psi(1+\epsilon)} \qquad \text{and} \qquad q_r^*(\psi) = \frac{1}{2} \cdot \frac{1}{1+\psi(1-\epsilon)}.$$

For any $\epsilon < 1$, **both state prices are decreasing in green supply ψ.** Hence **one is of the wrong sign.**

(Only exception is $\epsilon = 1$, in which case we recover Arrow securities.)

Control variables

Controlling for cross-asset spillovers?

One proposed solution to spillover biases is the use of controls, such as common factor exposures.

If spillovers occur mainly among similar assets, we should control for asset similarity.

This changes the unit of analysis to *residual cash flows* (rather than the asset).

Assets might be substitutable precisely because they have common factor exposures.

Problem: residual cash flow elasticities may be uninformative about asset-level elasticities.

Decision problem with traded factors

In our setting, the aggregate income in a given state is a factor (there are of course others).

- For example, the green asset has loadings $1+\epsilon$ and $1-\epsilon$ on state g and r income.
- To allow non-factor variation, perturb the model with small idiosyncratic noise, $y_j' = y_j + \eta_j$.

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Given this structure, we can model portfolio choice as a two-step problem:

1. Choose desired factor exposures (i.e. state-contingent consumption c_z) at price q_z.
2. Given c_z, choose how much idiosyncratic asset exposure $\tilde{w}_j$ to take on at price $\tilde{p}_j$.

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Controlling for factor exposures means focusing on the second step: **a conditional decision problem**.

Substitution across assets is now driven only by the idiosyncratic component. **A residual elasticity.**

Factor and residual elasticities

Consider a small perturbation, $Var(\eta_j) \approx 0$. Then optimal factor demand in portfolio shares is

$$\frac{c_z q_z}{W} = \pi_z.$$

The factor-level demand elasticity is zero (and thus very different from the asset elasticity.)

Fix factor exposures at 1 (as in the baseline model). **The elasticity of residual demand is**

$$\frac{\partial \tilde{\omega}_j^*}{\partial \tilde{p}_j} \frac{\tilde{p}_j}{\tilde{\omega}_j} = 1.$$

Highly non-linear: underlying elasticities are low even when the asset-level elasticity is very high.

Relative elasticities

An alternative: measuring relative elasticities

- Haddad et al. (2025): under slightly weaker symmetry, one can identify the "relative elasticity." i.e., the own-price minus the cross-price elasticity.

- Clarifies the identification challenge and partially circumvents it using a different estimand.
- Addresses it by decomposing elasticity matrix into an unobservable piece and one linked to factors.

- Our baseline model satisfies their assumptions. But small perturbations can still lead to biases.
- More generally, the symmetry assumption generically requires controls.

Are demand elasticities structural?

Counterfactuals from demand systems

Growing interest in using demand systems for counterfactuals and policy.

- Monetary policy transmission, FX interventions, QE, ...

Should demand elasticities be interpreted as “deep” parameters?

- Require invariant parameters to appropriately inform policymakers.

I. Dynamic trading

Except in special cases, financial assets are **investment goods**.

Investor demand depends on both own prefer and expected market returns (i.e., others' tastes).

$\Rightarrow$ Observed demand elasticities alone cannot identify *whose* tastes affect current demand.

But, for many counterfactuals we do need to be able to attribute tastes to investors.

One can generically rationalize a portfolio by constraints **or** preferences.

Problem: counterfactuals are generally sensitive to the precise micro-foundation.

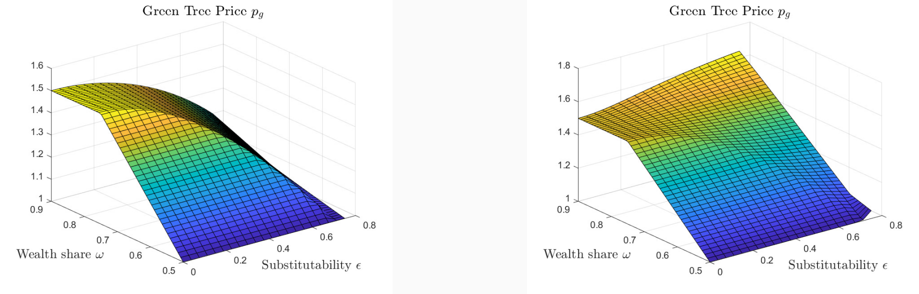

Figure 4: Equilibria in an economy with taste differences and one with portfolio restrictions on green asset share.

Conclusion

We study the methodological foundations of demand-based asset pricing, relying on principles of portfolio choice and equilibrium price determination.

1. Tastes may invalidate the organizing principle of no arbitrage.
2. Price spillovers offer a simple explanation for low *measured* elasticities.
3. Demand elasticities are structural only under stringent restrictions.

. . . Lots of work to be done.