A Trilemma for Asset Demand Estimation

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2026 WFA

Motivation

Influential literature aiming to estimate **demand functions** for financial assets from portfolio data.

- Promise: better understand price formation, policy transmission, counterfactuals.

Follows the **Marshallian paradigm**: measure elasticities (partial derivatives) of asset demand functions:

$$\mathcal{E}_{jk} = -\frac{\partial a_j(\vec{a}_{-j}, \vec{p})}{\partial p_k} \times \frac{p_k}{a_j(\vec{a}_{-j}, \vec{p})}.$$

Premise: elasticities are **stable, primitive empirical objects** which help distinguish competing theories.

This paper

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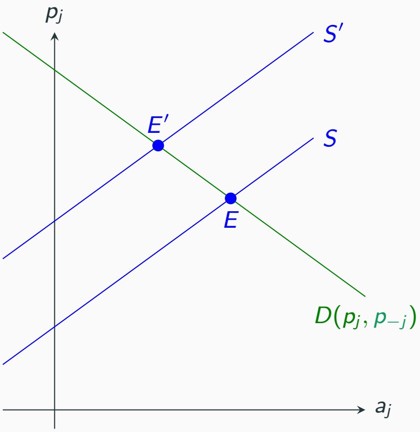
A: Not in the existing asset pricing paradigm – (i) deep preferences over payoffs and (ii) no arbitrage.

- Asset demand is derived from fundamental preferences for payoffs and a *mapping* into asset portfolios.
- Asset attributes (i.e. payoffs) are latent and vary $\rightarrow$ the mapping is fundamentally unidentified and unstable.

Elasticities are not portable data moments, they are inseparable from auxiliary theoretical assumptions.

- Beyond the usual "we need structure" – the structure itself cannot be fully diagnosed.

The Marshallian graph: ideal versus arbitrage-free asset markets



(a) Ideal: Move along **stable** demand curves.

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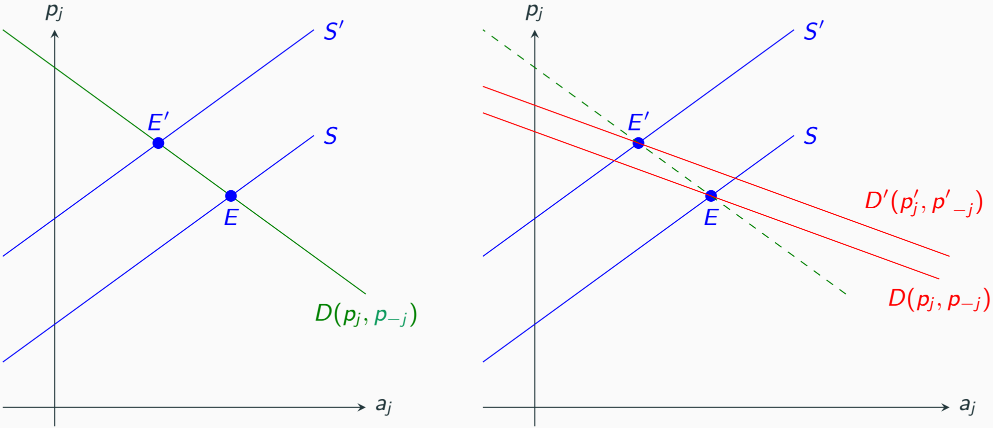

(a) Ideal: Move along **stable** demand curves.

(b) Demand generically shifts; **how is irreducibly theory**.

Framework

A paradigmatic asset pricing model

- Two dates, $t = 0, 1$. At date 1, one of Z states is realized.

- Investors choose among J assets. Y is the $J \times Z$ payoff matrix, $y_j(z) \geq 0$ is j's payoff in state z.
- **Markets can be complete or incomplete. Y is unobserved by the econometrician, may change.**

- p is the vector of asset prices, q is the vector of state prices (need not be unique).

- Potentially heterogeneous investors indexed by i.
- Investor i has endowment e^i_j of asset j. Aggregate endowment of asset j is $E_j = \sum_i e^i_j$.

- Equilibrium prices determined by market clearing.

Horn 1: no arbitrage

We consider the classical notion of no arbitrage.

Theorem (Fundamental Theorem of Asset Pricing)
There is no arbitrage if and only if there exist state prices $q \in \mathbb{R}^Z_{++}$ *such that*

$$p = Yq.$$

Benefits: Consistent pricing system, smooth demand functions, dimension reduction, consistent counterfactuals

Horn 2: preferences over payoffs

Start from the standard decision problem:

$$U^i = \sup_{a^i \in \Phi^i(p)} \quad u^i(c_0^i) + \delta^i \sum_{z=1}^{Z} \pi_z u^i(c_z^i) \qquad \text{(Utility over payoffs + constraints)}$$

$$\text{s.t.} \quad c_0^i = e_0^i - \sum_{j=1}^{J} p_j (a_j^i - e_j^i) \qquad \text{(Budget constraint)}$$

$$c_z^i = \sum_{j=1}^{J} y_j(z) a_j^i + w^i(z) \quad \text{for all } z \qquad \text{(Endowments= demand shifters)}$$

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Standard ID problem: use observed behavior to recover utility parameters and demand shifters. But:

1. **Indirect demand**: preferences pertain to *payoffs*, observations to *assets*. But Y and π are latent.
2. **Cross-asset linkages**: asset demand $a_j^i(\bar{p}, Y)$ generically depends on **all prices and payoffs.**

Step 1: Properties of Asset Demand

Asset demand: a decomposition

By the budget constraint, the consumption process induced by a portfolio a is

$$c = Y^{\mathrm{T}} a + w^i.$$

Hence a feasible target consumption vector c^* requires the portfolio:

$$a^*(c^*) = (Y^+)^{\mathrm{T}}(c^* - w^i)$$

By no arbitrage, $p = Yq$ (and so, using Moore-Penrose, $q = Y^+ p$). The chain rule then yields

$$-\frac{\partial a^*(c^*)}{\partial p^{\mathrm{T}}} = (Y^+)^{\mathrm{T}} \left(-\frac{\partial c^*}{\partial p^{\mathrm{T}}} \right) = (Y^+)^{\mathrm{T}} \underbrace{\left(-\frac{\partial c^*}{\partial q^{\mathrm{T}}} \right)}_{\equiv \mathcal{D}^i} \underbrace{\left(\frac{\partial q^{\mathrm{T}}}{\partial p^{\mathrm{T}}} \right)}_{= Y^+ \text{by NA}}.$$

Why is this decomposition useful?

$$\underbrace{-\frac{\partial a^i(p, Y, \text{shifters})}{\partial p}}_{\text{Asset demand}} = \underbrace{(Y^+)^T}_{\text{Mapping of } c \text{ into assets}} \left[\underbrace{-\frac{\partial c^i(q, \text{shifters})}{\partial q^{\mathrm{T}}}}_{\text{Fundamental payoff demand } \mathcal{D}^i} \right] \underbrace{Y^+}_{\text{Mapping of asset prices to state prices}} .$$

Fundamental payoff demand $\mathcal{D}^i \equiv -\frac{\partial c^i}{\partial q^{\mathrm{T}}}$ is **recognizable as a structural demand function.**

- It reflects internal preference parameters, not "external" characteristics of the choice set.
- It is invariant with respect to standard perturbations – depends on Y only through the asset span.

We can then understand properties of asset demand by analyzing the mapping through Y^+.

Proposition: Y^+ is fundamentally unobservable

Proposition (Non-identification of Y^+)

Consider any finite sample of realized payoffs. Then there exist arbitrarily many candidate payoff matrices Y that are observationally equivalent but have different generalized inverses.

Proof sketch. Take a candidate payoff matrix. Hold realized states fixed and perturb the payoff vector in a latent state z^* that never realizes in sample. Historical returns are unchanged, but Y^+ changes $\square$

Implications for asset demand

Asset demand is derived from fundamental demand using an **unobservable mapping** Y^+. This means:

1. Payoff demand $\mathcal{D}^i$ is **fundamentally unidentified** from *any* portfolio and price data.
   - i.e., there always exist multiple combinations of preferences and payoffs consistent with a given elasticity.
2. If $\mathcal{D}^i$ is structural with respect to perturbations of Y^+, then $\mathcal{A}^i = (Y^+)^{\mathrm{T}} \mathcal{D}^i Y^+$ is not.
   - Perturbations of Y^+ are *routine* in asset markets, including for standard policies and shocks.
   - Because the inverse payoff matrix matters, changes in payoffs to *any* asset matter.

Assumption of fixed Y^+ it is unverifiable from any finite data set $\rightarrow$ *assumed* Y pins down estimates.

Over-identification fails because the system is non-invertible with respect to a single "structural error."

Step 2:

Identification from Supply Shocks

If fundamental payoff demand cannot be learned, can I still learn asset demand?

Say we give up on identifying structural payoff demand $\mathcal{D}^i$. Can we identify asset demand $\mathcal{A}^i$?

We investigate this using: (i) perfectly exogenous asset-level supply shocks, and (ii) many shocks.

Answer: no, unless the assets are *Arrow securities* or payoffs do not change.

Single-asset supply shocks work only for Arrow securities

To identify demand for asset j, we need the state-price movement generated by a **pure change in** p_j:

$$\textbf{Under no arbitrage, } q = Y^+ p \textbf{ and so:} \qquad \Delta q_j^{\text{ideal}} = \frac{\partial q}{\partial p_j} = Y^+ e_j.$$

A supply shock instead changes state prices through the asset's payoff vector:

$$\textbf{Supply shock:} \qquad \Delta q_j^{\text{supply}} = \frac{\partial q}{\partial E_j} = -V y_j^{\mathrm{T}},$$

where V summarizes equilibrium state-price responses.

- The ideal experiment runs through the **inverse payoff mapping** Y^+.
- A supply shock runs through the asset's **payoff vector** y_j.
- These **directionally coincide** only in the Arrow-security case: assets without overlapping payoffs.

Case 2: How about multiple supply shocks?

Can we overcome the misalignment problem using many independent supply shocks?

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Proposition (Necessary conditions for identification of $\mathcal{A}^i$)

Generically, $\mathcal{A}^i$ is identified only if the econometrician:

1. Observes $K = J$ linearly independent experiments; and
2. Maintains the assumption that Y^+ is **fixed** across all experiments.

The fixed-Y^+ condition is central: it is unverifiable and forbids revisions in beliefs or future payoffs.

Payoff revisions are routine $\Rightarrow$ Identification relies on stringent assumption that cannot be tested.

Step 3: Mapping Y^+ is ill-conditioned

OK, but don't we know something about payoffs?

You may think: yes, Y is unknown, but it's not *that* unknown.

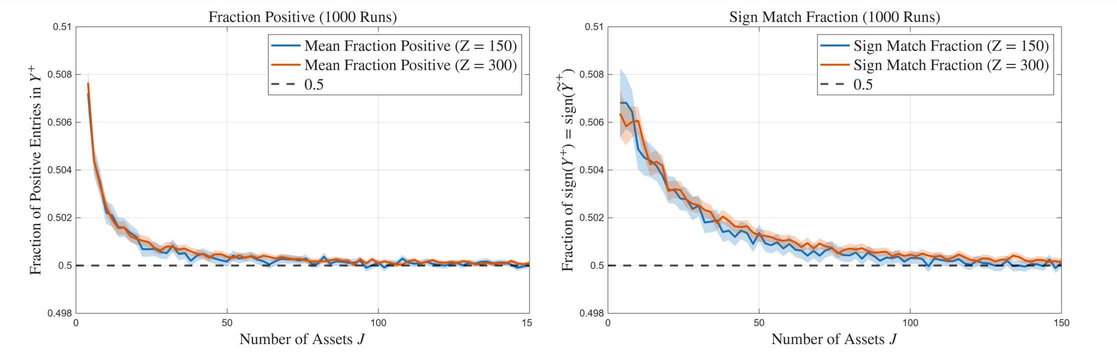
- We know about factor structures, estimated covariance matrices,...

Are statistical assumptions on Y enough to ensure well-behaved Y^+?

Answer from random matrix theory: no.

- Assume factor structure of Y, study asymptotic behavior of Y^+. Find: **signs of Y^+ are random**.
- Monte Carlo analysis confirms the same even for small asset menus.

Monte Carlo: sign instability converges fast



Left: fraction of positive entries in Y^+ converges to $\frac{1}{2}$ quickly.

Right: sign-match frequency between Y^+ and $\tilde{Y}^+$ (same factor model, independent idio. shocks).

Conclusion

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Within the predominant paradigm of asset pricing — (i) no arbitrage and (ii) preferences over payoffs — asset demand elasticities are theoretical objects:

- they cannot adjudicate between models, because they are **inseparable from the assumed structure**;
- they are structural only under the untestable assumption that payoffs or beliefs are never updated.

Implications: focus on robust moments that vary less with Y^+, or change the paradigm.

Appendix

Sign instability for realistic payoff processes

Theorem (Sign Instability of Y^+). Under a factor structure $y_{j,z} = \alpha_j + \beta_j f_z + \varepsilon_{j,z}$ and mild regularity conditions, for almost every realization of factor loadings:

(i) **Individual coin flip.** For each fixed asset j and state z, the sign of an element is a **coin flip:**

$$\lim_{J \to \infty} P\big((Y^+)_{z,j} > 0\big) = \frac{1}{2}.$$

(ii) **Knowledge of factors is insufficient.** Let Y and $\tilde{Y}$ share factor loadings and factor realizations but have independent idio. shocks. Then sign-determining variables are asymptotically independent:

$$q(J) \equiv \frac{1}{JZ} \sum_{j,z} \mathbf{1}\Big(\text{sign}(Y^+)_{z,j} = \text{sign}(\tilde{Y}^+)_{z,j}\Big) \to \frac{1}{2}.$$